

COMBINATORICS

THEOREMS & EXERCISES

1. MULTIPLICATION PRINCIPLE (FUNDAMENTAL PRINCIPLE OF COUNTING)

Suppose an event E can occur in m different ways and associated with each way of occurring of E , another event F can occur in n different ways, then the total number of occurrence of the two events in the given order is $m \cdot n$.

2. ADDITION PRINCIPLE

Suppose an E occurs in m different ways and associated with each way of occurring of E , another event F can occur in n different ways, then the total number of occurrence of the one of two events is $m + n$.

3. PERMUTATIONS

The number of possibilities to arrange n (distinguishable) objects in a row (so-called permutations) is **n -factorial**.

$$n! = n \cdot (n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1.$$

4. SEQUENCES (VARIATIONS WITH REPETITIONS)

There are n^k different sequences of length k that can be formed from elements of a set X consisting of n elements (elements are allowed to occur several times in a sequence).

5. SEQUENCES (VARIATIONS) WITHOUT REPETITIONS

The number of sequences of length k without repetitions whose elements are taken from a set X comprising n elements is

$$= \frac{n!}{(n-k)!} = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-k+1)$$

6. COMBINATIONS (CHOOSING A SUBSET) The number of possibilities to choose a subset of k elements from a set of n elements (the order being irrelevant) is **n choose k**

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Problem 1 *How many Lotto-combinations (6 numbers out of 1, 2, . . . , 49) contain two consecutive numbers?*

Problem 2 *King Arthur chooses three of the 25 knights sitting around his table to fight a fearsome dragon. How many possible choices are there, if no two of the chosen knights should sit next to each other?*

Problem 3 *How many possible ways are there to form five-letter words using only the letters A,B,C,D,E,F,G,H? How many such words are there that do not contain a letter twice?*

Problem 4 *In how many possible orders can the letters of the word MATHEMATICS be arranged?*

Problem 5 *At a sokkie, there are 20 girls and 20 boys. How many ways are there to form 20 pairs?*

Problem 6 *At a certain university, all second-year science students may choose either mathematics, or physics, or both. The mathematics course is attended by 50 students, the physics course by 30 students. 15 students attend both courses. How many second-year science students are there?*

Problem 7 *Third-year science students also have the opportunity to attend chemistry, but every student has to take at least one of the three courses. Altogether, there are 40 students in the mathematics class, 25 who attend physics, and 20 who attend chemistry. Furthermore, we know that 10 students do both mathematics and physics, 8 both mathematics and chemistry, and 7 physics and chemistry. There are two particularly keen students who attend all three courses. How many third-year science students are there?*

Problem 8 *How many possibilities are there to distribute 15 letters among 5 people so that each of them receives at least one letter?*

Problem 9 *From 10 people, how many ways can you form a committee of 7 people consisting of a president, two (equivalent) vice presidents, and four (equivalent) regular members?*

SOURCES:

1. <http://math.sun.ac.za/swagner/Combinatorics.pdf>
2. Edward A. Bender S. Gill 'Williamson Foundations of Combinatorics with Applications'