1.2 SETS, INTERVALS
THEORY

1. SETS

   (a) Basic Information

   A set (or **class**) is an **unordered collection of objects**, which are arranged in a group, The set with any numbers use the symbol braces $\{\}$, and will be denoted by Capital letters $A, B, C, \ldots$.

   The objects in a set are called **the elements**, or **members of the set**. A set is said to **contain its elements**. The objects comprising the set are called its elements or members and will be denoted by lower case letters $a, b, c, \ldots$. We write $a \in X$ when a is an element of the set X . we read a $a \in X$ as a **is a member of** X or a **is an element of** X or a **belongs to** X.

   For describing sets there are two ways of describing, or specifying the members of, a set.

   - **by using a rule** or semantic description:
     $S = \{x : x \in \mathbb{Z} \wedge\ 5 < x < 15\}$ — which reads S is the set of x such that x is an integer and x is greater than 5 and less than 15.
   - **by extension** – that is, listing each member of the set. An extensional definition is denoted by enclosing the list of members in curly brackets:
     $C = \{4, 2, 1, 3\}$, $D = \{\text{white}, \text{black}, \text{red}, \text{green}\}$.

   Definition 1 *The universal set U is the set containing everything currently under consideration. or all the sets under consideration will likely to be subsets of a fixed set called **Universal Set**.*

   Definition 2 *A set which has no element is called **the null set** or **empty set** and is symbolized by $\emptyset$.*

   (b) Subsets and Set Equality

   Definition 3 *A Set **A is a subset of set B** if every element of A is also an element of B.*

   $$A \subseteq B \Leftrightarrow \forall x \qquad x \in A \Rightarrow x \in B$$

   Definition 4 *Two sets A and B are **equal** if they have the same elements.*

   $$A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A$$

   Definition 5 *: A is **a proper subset** of B if $A \subseteq B$ and $A \neq B$. This is denoted by $A \subset B$.*

   $$A \subset B \Leftrightarrow \forall x(x \in A \Rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$

   (c) Set Operations

   Definition 6 *The **union of two sets** A and B denoted $A \cup B$, is the set of all objects that are members of A, or B.*

   $$A \cup B = \{x : x \in A \vee x \in B\}$$

Definition 7 *The **intersection of two sets** A and B denoted $A \cap B$, is the set of all objects that are members of A, or B.*

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

Definition 8 *Two sets A and B are called **mutually exclusive** if their intersection is empty. Mutually exclusive sets are also called **disjoint**.*

$$A \cap B = \emptyset$$

General intersection of several sets: $A_1 \cap \ldots.. \cap A_n = \{x : x \in A_1 \wedge \ldots.. \wedge A_n\}$

Definition 9 ***The complement of a set A**, denoted by A^c, is the set of elements which belong to U but which do not belong to A.is defined by*

$$A^c = \{x : x \in U \vee x \notin A\}$$

Definition 10 ***The difference between sets A and B**, denoted $A - B$ is the set containing the elements of A that are not in B.*

$$A - B = \{x : x \in A \wedge x \notin B\} = A \cap B^c$$

*$A - B$ is also called **the complement of B with respect to A** (relative complement.)*

Similarly $B - A = \{x : x \in B \wedge x \notin A\} = B \cap A^c$

Definition 11 ***The symmetric difference between sets A and B**, denoted $A \oplus B$ is the set containing the elements of A that are not in B or vice-versa.*

$$A \oplus B = (A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

(d) Algebra of sets

- **Idempotence:** Union and intersection of a set with itself are
  $A \cup A = A$
  $A \cap A = A$
  Associativity: If we have three sets A, B and C, then
  $(A \cup B) \cup C = A \cup (B \cup C)$
  $(A \cap B) \cap C = A \cap (B \cap C)$

- **Commutativity:** Union and intersection of two sets are commutative. Hence,
  $A \cup B = B \cup A$
  $A \cap B = B \cap A$

- **Distributivity:** In set theory, we have two distribution laws as
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- **Identity:** If $\emptyset$ is an empty set, A is any given set and U is universal set then:
  $A \cup \emptyset = A$
  $A \cap U = A$
  $A \cup U = U$
  $A \cap \emptyset = \emptyset$

- $A \cup A^c = U$
  $A \cap A^c = \emptyset$
- $U^c = \emptyset$
  $\emptyset^c = U$
- $(A^c)^c = A$

- **De-Morgan's laws:**
  $(A \cup B)^c = A^c \cap B^c$
  $(A \cap B)^c = A^c \cup B^c$

2. INTERVALS

   (a) **Proper and bounded:**
   Open: $(a, b) = \{x : a < x < b\}$
   Closed: $[a, b] = \{x : a \leq x \leq b\}$
   Left-closed, right-open: $[a, b) = \{x : a \leq x < b\}$
   Left-open, right-closed: $(a, b] = \{x : a < x \leq b\}$

   (b) **Left-bounded and right-unbounded:**
   Left-open: $(a, +\infty) = \{x : x > a\}$
   Left-closed $[a, +\infty) = \{x : x \geq a\}$

   Left-unbounded and right-bounded:

   Right-open: $(-\infty, b) = \{x : x < b\}$
   Right-closed: $(-\infty, b] = \{x : x \leq b\}$

   (c) **unbounded at both ends (simultaneously open and closed)**: $(-\infty, +\infty) = \mathbb{R}$