2. Algebraic expressions, equations, inequalities

Task 2.01. (0-4) (2015 - task 16)

Write down each of the following as an algebraic expression.

a) the cube of the sum of a and b.

....

b) the difference of a squared and b squared

....

c) the quotient of the absolute value of a doubled and b

…... .

d) the cube root of the absolute value of the quotient of a and b ....

.

Solution 2.01. B

2.01 a) $(a+b)^3$ b) a^2-b^2 c) $|a| \div (2b)$ d) $\sqrt[3]{|a \div b|}$

Task 2.02. (0-1) (2016 - task 04)

If $m=\frac{1-x^2}{x+1}$, $n=x-1$, where $x \neq -1$ then the difference between m and n equals

A. 0 B. $2-2x$ C. $-2x$ **D.** $\frac{-x^2-x+2}{x+1}$

Solution 2.02. B

$$m-n=\frac{1-x^2}{x+1}-(x-1)=\frac{(1-x)(1+x)}{x+1}+1-x=1-x+1-x=2-2x$$

2. Algebraic expressions, equations, inequalities

Task 2.03. (0-2) (2016 - task 19)

The equation $mx^2 + 2x - 1 = 0$ is solved for x. Complete the following sentences.

(a) If $m = -1$, then the number of solutions to this equation is ....

(b) If the number $x_0 = \frac{1}{2}$ is the solution to this equation then $m =$........................... .

Solution B 2.03. (a) one (b) **0**

(a) For $m = -1$ then the equation $mx^2 + 2x - 1 = 0$ has the form

$$-x^2 + 2x - 1 = 0.$$

We can multiply both sides of the equation by (-1) :

$$-x^2 + 2x - 1 = 0 \qquad / \times (-1)$$

$$x^2 - 2x + 1 = 0$$

The left side of the last equation is a square of the difference:

$$(x - 1)^2 = 0$$

and so the equation has **one** solution (which is $x = 1$).

(b) If $x_0 = \frac{1}{2}$ is the solution to the equation $mx^2 + 2x - 1 = 0$ then

$$m \times \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} - 1 = 0$$

$$\frac{1}{4}m + 1 - 1 = 0$$

$$m = 0$$

Task 2.04. (0-1) (2017 - task 03)

If $m = 5$ and $n = 4$, then the difference of squares of m and n is:

A. 41 B. 1 C. 81 D. 9

Solution 2.04 D

$m^2 - n^2 = 5^2 - 4^2 = 25 - 16 = 9$

2. Algebraic expressions, equations, inequalities

Task 2.05. (0-1) (2017 - task 04)

$\begin{cases} 2x + y = 3 \\ 4x - 5y = -1 \end{cases}$ is a system of equations which is represented in two-dimensional system of coordinates by:

A. an infinite set

B. an empty set.

C. exactly two distinct points.

D. exactly one point.

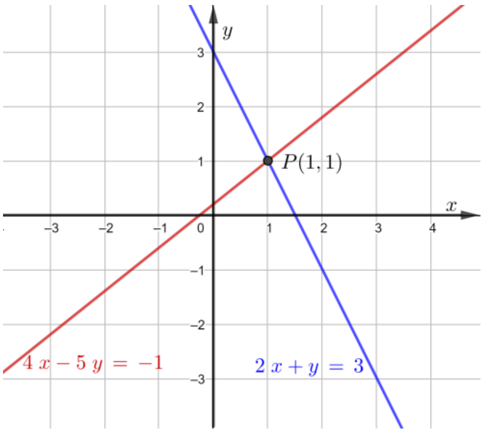
Solution 2.05 D

This system of equations is represented by exactly one point $P(1{,}1)$ which is a common point of two lines given by equations $2x + y = 3$ and $4x - 5y = -1$.

You can find this point algebraically, for example:

$\begin{cases} 2x + y = 3 \\ 4x - 5y = -1 \end{cases}$ $\begin{cases} y = 3 - 2x \\ 4x - 5(3 - 2x) = -1 \end{cases}$ $\begin{cases} y = 1 \\ x = 1 \end{cases}$ $\begin{cases} x = 1 \\ y = 1 \end{cases}$

or by graphing the lines:



Task 2.06. (0-1) (2017 - task 05)

The sum of all roots of the equation $(x - 3)(x - 2)(x + 6) = 0$ is:

A. -1

B. 1

C. -11

D. 11

Solution 2.06 A

The product is equal to 0 if and only if at least one of the factors is equal to 0.

Thus the roots of the given equations are 3, 2 and -6 .

The sum of the roots is $3 + 2 + (-6) = -1$.

2. Algebraic expressions, equations, inequalities

Task 2.07. (0-1) (2018 – task 11)

The set of all real numbers x which satisfy the inequality: $-3 < 2x - 1 < 3$ is

A. $(-3; 3)$ **B.** $\langle -3; 3\rangle$ **C.** $(-1; 2)$ D. $\langle -1; 2\rangle$

Solution C

$-3 < 2x - 1 < 3$ $/+1$

$-2 < 2x < 4$ $/\div 2$

$-1 < x < 2$

$x \epsilon (-1, 2)$

Task 2.08. (0-1) (2019 – task 02)

For each real number *x* and for each real number *y* the square of the difference $(x^2 - 5y)^2$ equals:

A. $x^4 - 10x^2y + 25y^2$ **B.** $-x^4 + 10x^2y - 25y^2$

C. $x^4 + 25y^2$ **D.** $x^4 - 25y^2$

Solution 2.08. A

We can apply the short multiplication formula $(a - b)^2 = a^2 - 2ab + b^2$:

$(x^2 - 5y)^2 = (x^2)^2 - 2x^2 5y + (5y)^2 = x^4 - 10x^2y + 25y^2$

Task 2.09. (0-1) (2019 – task 03)

The set of simultaneous equations $\begin{cases} 3x + 5y = -1 \\ x - 11y = 6 \end{cases}$ in a set of coordinate axes:

A. describes an infinite set. **B.** describes an empty set.

C. describes exactly two distinct points. **D.** describes exactly one point.

Solution 2.09. D

The given set of equations contains equations of two straight lines $3x + 5y = -1$ and $x - 11y = 6$, which are **not parallel** because they have different gradients: gradient of the first line is $\frac{-5}{3}$ and the gradient of the second line is $\frac{1}{11}$. So this set describes exactly one point, which is common for two lines.

2. Algebraic expressions, equations, inequalities

Task 2.10. (0-1) (2020 – task 02)

The product of all solutions of the equation $(x-1)(x+2)(x-3)=0$ is:

A. -6 B. -2 C. 2 D. 6

Solution 2.10 A

The product is equal to 0 if and only if at least one of the factors is equal to 0.

Thus the solutions of the given equations are 1, -2 and 3 .

The product of the solutions is $1 \times (-2) \times 3 = -6$.

Task 2.11. (0-1) (2020 – task 03)

If $x + y = 25$ and $x - y = -4$, then $x^2 - y^2$ equals:

A. -100 B. -29 C. 29 D. 100

Solution 2.11 A

$x^2 - y^2 = (x+y)(x-y) = 25 \times (-4) = -100$

Task 2.12. (0-1) (2020 – task 08)

The expression $2(x-3) - 5x\,(3-x)$ can be written as:

A. $-10x(x-3)$	**B.** $10x(x-3)$
C. $(5x-2)(x-3)$	**D.** $(5x+2)(x-3)$

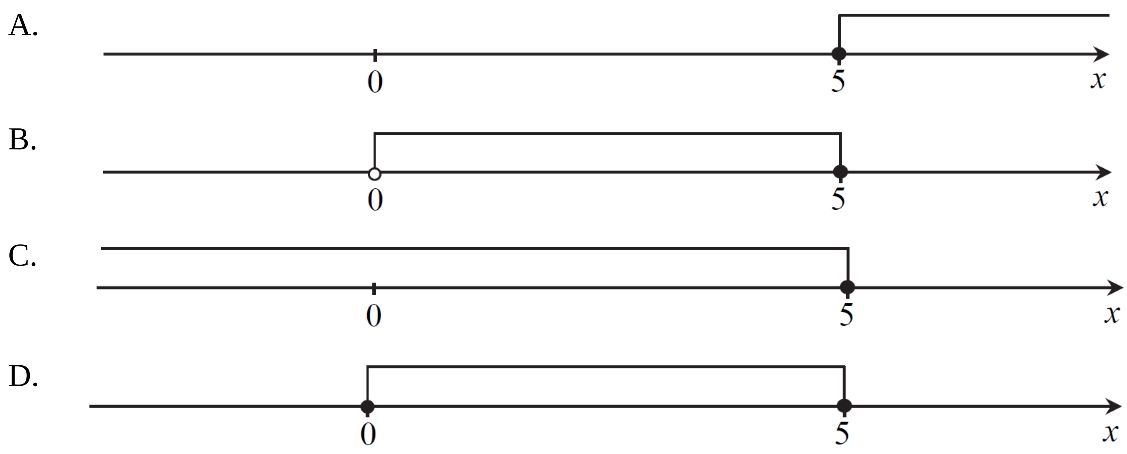
Solution 2.12 D

$2(x-3) - 5x\,(3-x) = 2(x-3) + 5x\,(x-3) = (x-3)(2+5x)$

2. Algebraic expressions, equations, inequalities

Task 2.13. (0-1) (2020 – task 09)

The solution set for the inequality $2-\frac{2}{3}(x-1) \geq -\frac{2}{3}$ is the interval:



Solution 2.13. C

$2-\frac{2}{3}(x-1) \geq -\frac{2}{3}$

$2-\frac{2}{3}x+\frac{2}{3} \geq -\frac{2}{3}$ $\quad /\times 3$

$6-2x+2 \geq -2$

$-2x \geq -10$ $\quad /\div(-2)$

$x \leq 5$

Task 2.14. (0-2) (2020 – task 18)

The geometrical interpretation of the set of simultaneous equations

$$\begin{cases} x+y=2 \\ x+(1+m)y=1 \end{cases}$$

with the unknowns x and y are:

(a) two parallel lines, when m equals ....

(b) two perpendicular lines, when m equals ....

2. Algebraic expressions, equations, inequalities

Solution 2.14. (a) $-\mathbf{1}$ (b) $-\mathbf{2}$

The first line given by the equation $x + y = 2$ has the gradient $a_1 = -1$.

The second line given by the equation $x + (1 + m)y = 2$ has the gradient $a_2 = \frac{-1}{1+m}$ for $m \neq 1$.

In the case when $m \neq 1$:

(a) the lines are parallel if and only if they have the same gradients: $-1 = \frac{-1}{1+m}$, and so $m = 0$.

(b) the lines are perpendicular if they product of their gradients is -1: $\frac{-1}{1+m} \times (-1) = -1$, which means that $1 + m = -1$ and so $m = -2$.

In case when $m = -1$ the second line has the equation $x = 1$.

The line $x = 1$ is not parallel nor perpendicular to the line $x + y = 2$.

Task 2.15. (0-1) (2021 – task 01)

The square of the difference of $3x$ and y, minus the square of the sum of x and $3y$ is

A. $8x^2 + 8y^2 - 12xy$ **B.** $8x^2 - 8y^2$

C. $8x^2 - 8y^2 - 12xy$ **D.** $8x^2 + 8y^2$

Solution 2.15. C

$(3x - y)^2 - (x + 3y)^2 =$

$= (9x^2 - 6xy + y^2) - (x^2 + 6xy + 9y^2) =$

$= 8x^2 - 8y^2 - 12xy$

Task 2.16. (0-1) (2021 – task 03)

The solution for the inequality

$$\frac{x-2}{2} - \frac{9-x}{3} > \frac{1}{6}x - 10$$

is

A. $(-9; +\infty)$ B. $\left(-\frac{36}{11}; +\infty\right)$ C. $\left(\frac{7}{2}; +\infty\right)$ D. $\boldsymbol{R}$

Solution 2.16. A

$\frac{x-2}{2} - \frac{9-x}{3} > \frac{1}{6}x - 10$ $/\times 6$

$3(x - 2) - 2(9 - x) > x - 60$

2. Algebraic expressions, equations, inequalities

$4x > -36 \qquad /\div 4$

$x > -9$

Task 2.17. (0-1) (2021 – task 04)

The greatest real root of the equation $x(x + 1)(3x + 4) = 0$ is

A. 1 B. 0 C. 2 **D.** $-\frac{4}{3}$

Solution 2.17. B

The product is equal to 0 if and only if at least one of the factors is equal to 0.

Thus the roots of the given equations are 0, -1 and $-\frac{4}{3}$.

The greatest of them is 0.

2. Algebraic expressions, equations, inequalities

Answers

2.01 a) $(a+b)^3$ b) a^2-b^2 c) $|a| \div (2b)$ d) $\sqrt[3]{|a \div b|}$

	2.02 B	2.03 (a) one (b) 0	2.04 D	2.05 D
2.06 A	2.07 C	2.08 A	2.09 D	2.10 A
2.11 A	2.12 D	2.13 C	2.14 (a)-1 (b)-2	2.15 C
2.16 A	2.17 B			