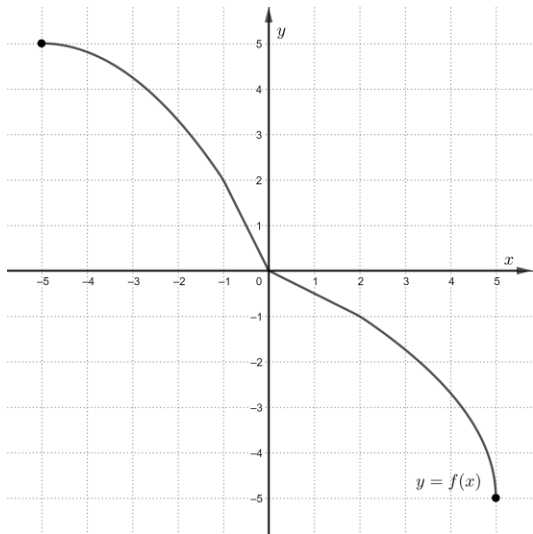


3. Functions

Illustration for tasks 3.1. – 3.4.



The illustration shows the graph of a function f .

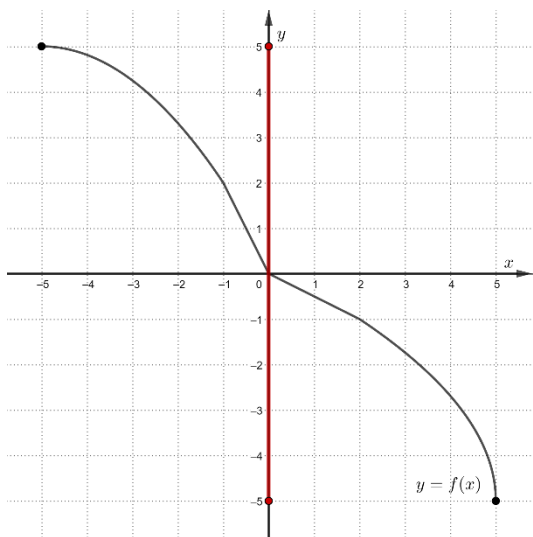
Task 3.01. (0-1) (2015 – task 01)

The range of the function f belongs to the following interval:

- A. $\langle -1; 2 \rangle$ B. $\langle 0; 5 \rangle$ C. $\langle -5; 5 \rangle$ D. $\langle -5; 0 \rangle$

Solution 3.01. C

The range of a function is the set of all its values. You read the values on y-axis.



Task 3.02. (0-1) (2015 – task 02)

The zero of the function f is

- A. $x = -5$ B. $x = 0$ C. $x = 2$ D. $x = 5$

Solution 3.02. B

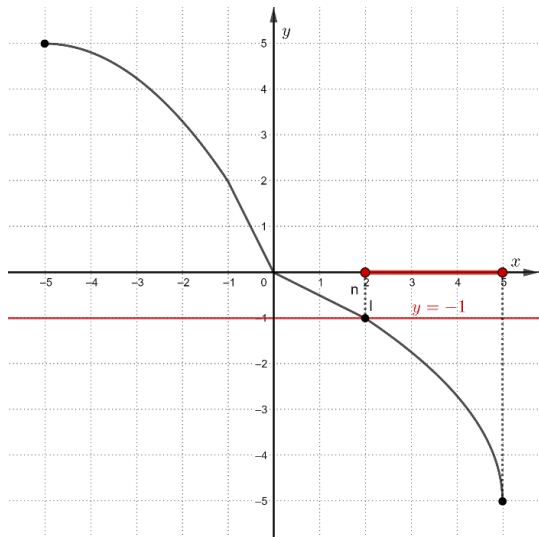
$f(x) = 0$ for $x = 0$

Task 3.03. (0-1) (2015 – task 03)

The set of solutions of inequality $f(x) \leq -1$ is the following interval:

- A. $\langle 2; 5 \rangle$ B. $\langle -5; 2 \rangle$ C. $\langle -5; 1 \rangle$ D. $\langle -1; 5 \rangle$

Solution 3.03. A

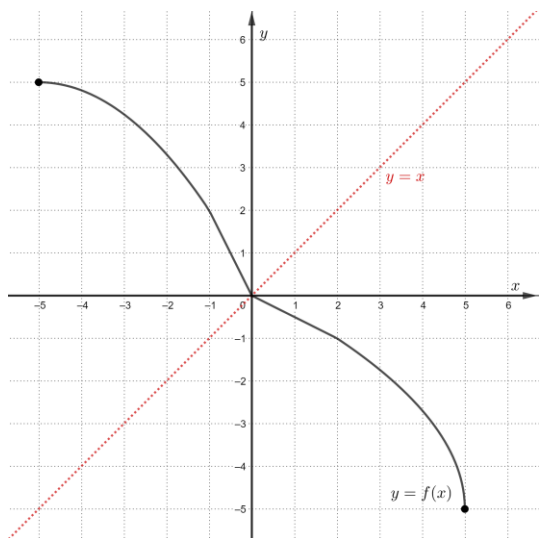


Task 3.04. (0-1) (2015 – task 04)

The graph of the function f is symmetrical about

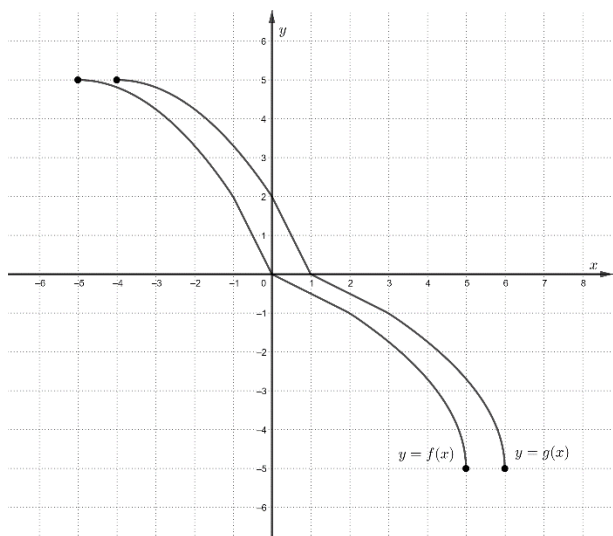
- A. the OX axis B. the OY axis C. the origin D. the line $y = x$

Solution 3.04 D



3. Functions

Task 3.05. (0-1) (2015 – task 05)



The function g was plotted by translating the graph of the function f along one of the axes of the coordinate system (see illustration). The function g can be expressed in the following way:

- A. $g(x) = f(x) - 1$
- B. $g(x) = f(x + 1)$
- C. $g(x) = f(x - 1)$
- D. $g(x) = f(x) + 1$

Solution 3.05 C

Task 3.06. (0-3) (2015 – task 14)

Given the function f with the formula $f(x) = -x^2 - 2x + 3$, complete the following sentences.

- (a) The function f reaches the maximum value of for x equal to
- (b) The value of the function for $x = -5$ is the same as for another x equal to
- (c) The function f has negative values if, and only if, the x values belong to the set

Solution 3.06 (a) **4**, **-1** (b) **-7** (c) **$(-\infty, -3) \cup (1, \infty)$** .

- The function $f(x) = -x^2 - 2x + 3$ is a quadratic function of the general form $f(x) = ax^2 + bx + c$ where $a = -1$, $b = -2$, $c = 3$.
- It's graph of the function is a **parabola with branches directed downwards** because the coefficient a is a negative number ($a < 0$).

- The determinant of the function is:

$$\Delta = b^2 - 4ac = (-2)^2 - 4 \times (-1) \times 3 = 16$$

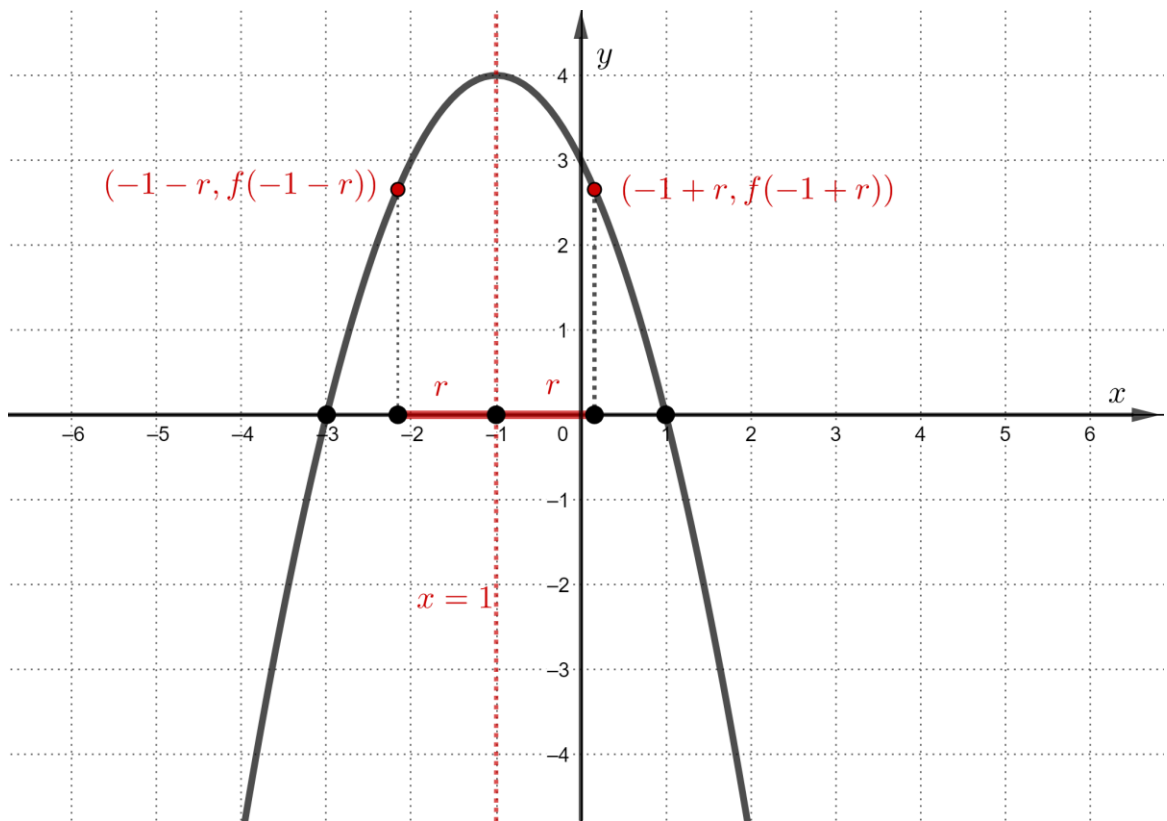
- Coordinates of the parabola vertex $W(p, q)$ are:

$$p = \frac{-b}{2a} = \frac{2}{-2} = -1,$$

$$q = \frac{-\Delta}{4a} = \frac{-16}{-4} = 4.$$

So the vertex of the parabola is the point $W(-1, 4)$.

- (a) The function reaches the maximum value $y = 4$ for x equal to -1 .
- (b) The line of symmetry has the equation $x = -1$, so for every for all real numbers $f(-1 - r) = f(-1 + r)$. So $f(5) = f(-1 + 6) = f(-1 - 6) = f(-7)$



- (c) The function has two zeros:

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2 + \sqrt{16}}{-2} = -3$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2 - \sqrt{16}}{-2} = 1$$

You can read the set of arguments for which the function has negative values from the graph: $f(x) < 0$ for $x \in (-\infty, -3) \cup (1, \infty)$.

3. Functions

Task 3.07. (0-1) (2016 – task 15)

The linear function $y = (3 - m)x + 6$ has no x -intercepts when

- A. $m = 3$ B. $m = 0$ C. $m = 6$ D. $m = -3$

Solution 3.07. A

Task 3.08. (0-1) (2016 – task 06)

The quadratic function f takes negative values for all arguments in the $(-2, 3)$ interval and for no other arguments. The solution set for the inequality $f(x - 3) < 0$ is the interval

- A. $(-5; 0)$ B. $(1; 6)$ C. $(-2, 3)$ D. $(-3; 2)$

Solution 3.08. B

$$f(x - 3) < 0$$

$$-2 < x - 3 < 3$$

$$1 < x < 6$$

Task 3.09. (0-2) (2016 – task 18)

The linear function f has the equation $f(x) = -\frac{1}{2}x + 13$.

Complete the following sentences.

- (a) For the argument -4 , the value of the function f equals
- (b) The x -intercept of the function equals
- (c)

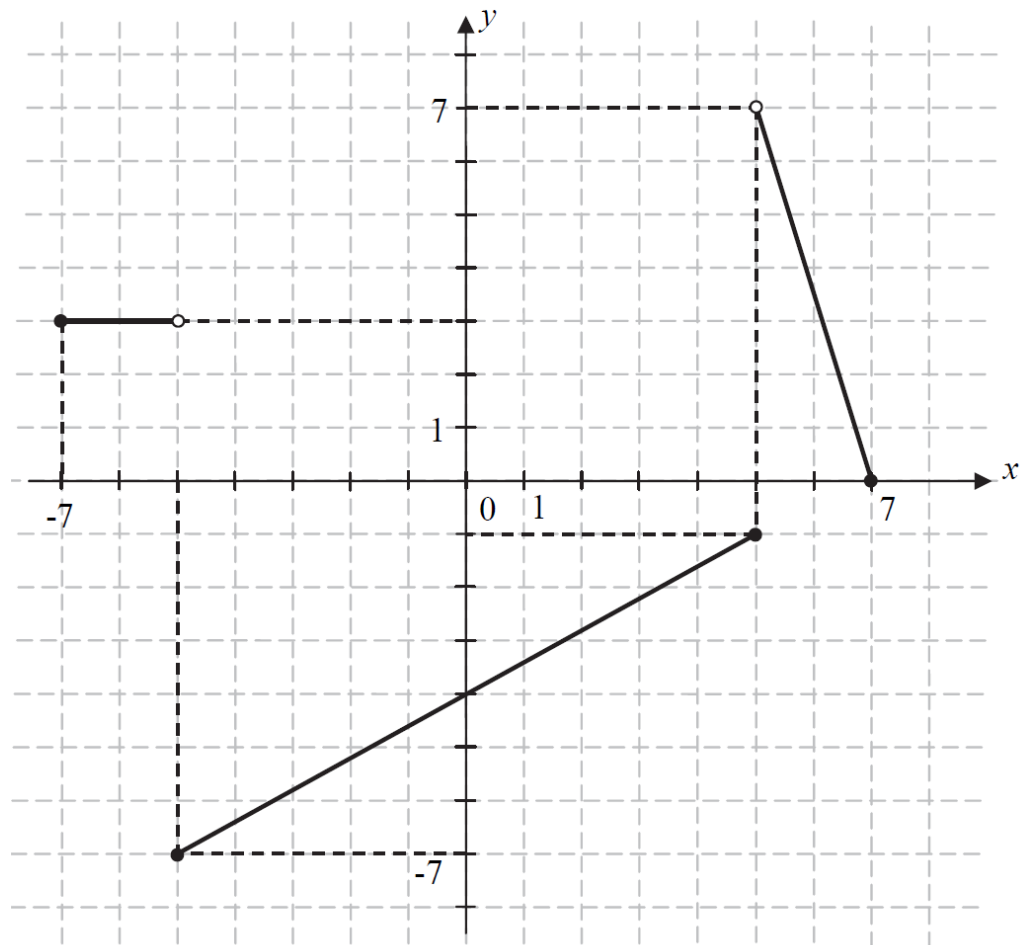
Solution 3.09. (a) 15 (b) 26

$$(a) f(-4) = -\frac{1}{2} \times (-4) + 13 = 2 + 13 = 15$$

$$(b) f(x) = 0 \text{ if and only if } -\frac{1}{2}x + 13 = 0$$

Task 3.10. (0-5) (2016 – task 20)

The following illustration shows the graph of the function f .



Complete the following sentences based on the illustration.

- (a) The domain of the function is the set $D = \dots\dots\dots$
- (b) The range of the function is $Z_w = \dots\dots\dots$
- (c) The longest interval in which the function f decreases is $\dots\dots\dots$
- (d) The lowest value of the function f equals $\dots\dots\dots$
- (e) The solution set for the inequality $f(x) < -1$ is $\dots\dots\dots$

Solution 3.10

- (a) $\langle -7, 7 \rangle$ (the closed interval)
- (b) $\langle -7, 7 \rangle$ (the interval left-closed and right-open)
- (c) $(5, 7)$ (the interval left-open and right-closed)
- (d) -7
- (e) $x \in (-5, 5)$

3. Functions

Task 3.11. (0-1) (2017 – task 06)

The graph of a linear function f is a line which crosses the axes of the coordinate system at $K = (-5, 0)$ and $L = (0, 7)$. Therefore, the equation of function f is:

A. $f(x) = -\frac{7}{5}x + 7$

B. $f(x) = \frac{7}{5}x + 7$

C. $f(x) = \frac{5}{7}x - 5$

D. $f(x) = -\frac{5}{7}x - 5$

Solution 3.11 B

The graph of the linear function

is a straight line given by slope-intercept equation $y = ax + b$.

- The coefficient a is the slope (gradient) of the line:

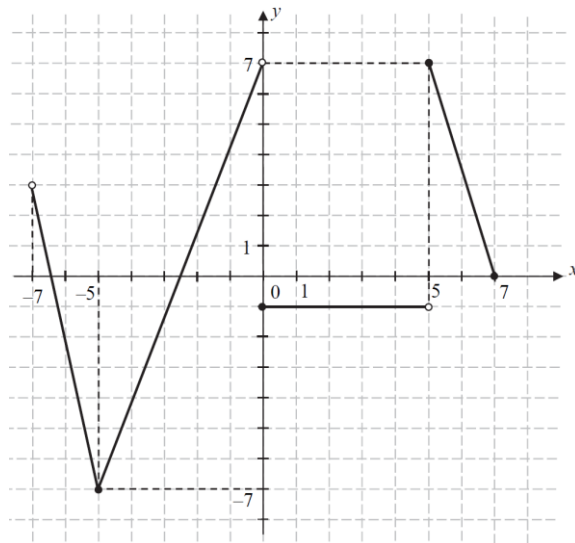
$$a = \frac{y_L - y_K}{x_L - x_K} = \frac{7 - 0}{0 - (-5)} = \frac{7}{5}.$$

- The b coefficient indicates the height at which the straight line intersects with the y -axis. The point $L = (0, 7)$ is on the y -axis, so:

$$b = 7.$$

Task 3.12. (0-5) (2017 – task 15)

The illustration shows the graph of a function f .



Complete the following sentences.

- (a) The domain of function f is the set $D = \dots\dots\dots$
- (b) The range of function f is $Z_w = \dots\dots\dots$
- (c) The maximum of function f equals $\dots\dots\dots$
- (d) The longest interval in which function f is increasing is $\dots\dots\dots$
- (e) The number of x -intercepts of function f equals $\dots\dots\dots$

Solution 3.12.

- (a) $(-7, 7)$ (the interval left-open and right-closed)
- (b) $\langle -7, 7 \rangle$ (the closed interval)
- (c) 7
- (d) $\langle -5, 0 \rangle$ (the interval left-closed and right-open)
- (e) 3

Task 3.13. (0-1) (2018 – task 05)

The exponential function f is given by the equation $f(x) = 2^x$. The graph of a function g is obtained by translating the graph of function f three units upwards along the axis OY . Hence, function g is defined by the equation

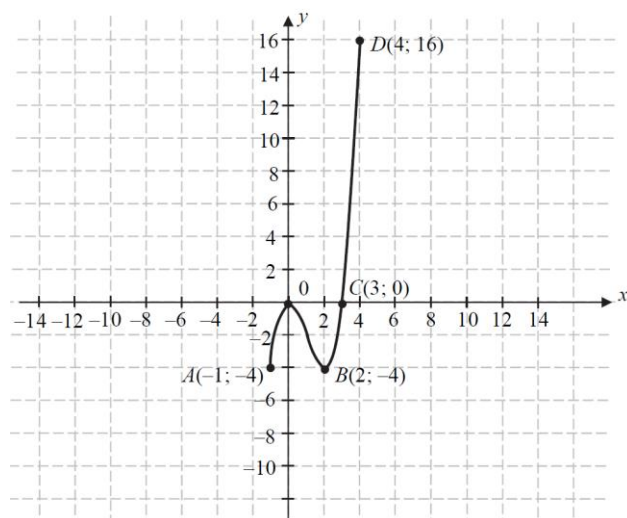
- | | |
|---------------------|---------------------|
| A. $g(x) = 2^{x+3}$ | B. $g(x) = 2^x + 3$ |
| C. $g(x) = 2^{x-3}$ | D. $g(x) = 2^x - 3$ |

Solution 3.13. B

3. Functions

Information for tasks 3.14 – 3.17.

The illustration shows the graph of a function f . It has two zeros which are both integers.



Task 3.14. (0-1) (2018 – task 06)

The domain of the function is the set:

A. $\langle -1; 16 \rangle$	B. $\langle -4; 16 \rangle$	C. $\langle -1; 4 \rangle$	D. $\langle -1; 3 \rangle$
------------------------------------	------------------------------------	-----------------------------------	-----------------------------------

Solution 3.14. C

Task 3.15. (0-1) (2018 – task 07)

The range of the function f is the set:

- A. $\langle -1; 4 \rangle$ B. $\langle -1; 3 \rangle$ C. $\langle 0; 16 \rangle$ D. $\langle -4; 16 \rangle$

Solution 3.15. D

Task 3.16. (0-1) (2018 – task 08)

Function f reaches its minimum for:

A. $x = 0$ and $x = 3$	B. $x = -1$ and $x = 2$
C. $x = -1$ and $x = -4$	D. $x = 2$ and $x = 4$

Solution 3.16. B

Task 3.17. (0-1) (2018 – task 09)

The zeros of the function f are the numbers

- A. 0 and 3 B. -1 and 2 C. -1 and -4 D. 2 and -4

Solution 3.17. A

Task 3.18. (0-3) (2018 – task 16)

A quadratic function f is given by the equation: $f(x) = 2x^2 - 8x - 10$. Complete the following sentences.

- (a) The interval in which the function is decreasing is
- (b) The range of the function is the interval:
- (c) The function assumes non-negative values if and only if the x arguments belong to the set

Solution 3.18.

$$f(x) = 2(x - 2)^2 - 18 = 2(x + 1)(x - 5)$$

- (a) $(-\infty, 2)$
- (b) $\langle -18, \infty$
- (c) $(-\infty, -1) \cup \langle 5, \infty$

Task 3.19. (0-1) (2019 – task 04)

The quadratic function f takes positive values for all x within the interval $(-8, 16)$ and for no other x . The solution set for the inequality $f(x + 4) > 0$ is the interval:

- A. $(-12; 20)$ B. $(-4; 20)$ C. $(-4; 12)$ D. $(-12; 12)$

Solution 3.19. D

$$f(x + 4) > 0$$
$$-8 < x + 4 < 16$$
$$-12 < x < 12$$

Task 3.20. (0-1) (2019 – task 05)

The four functions: f_1, f_2, f_3, f_4 are defined for all real numbers by the following formulas:

$$f_1(x) = x^2 - x + 2019, \quad f_2(x) = (x^2 + 2019)(x^2 + 1),$$
$$f_3(x) = -(x - 2019)(x^2 + 1), \quad f_4(x) = -x^2 + 11x - 2019$$

One of these functions has a zero. This function is:

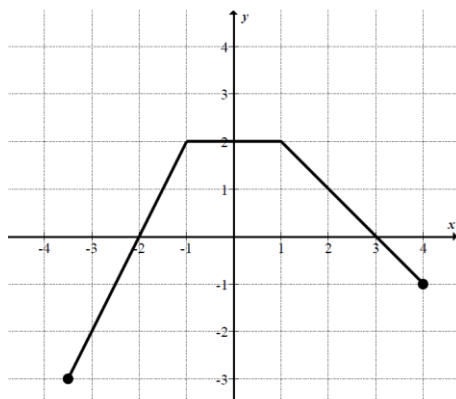
- A. f_1 B. f_2 C. f_3 D. f_4

Solution 3.20. C

3. Functions

Task 3.21. (0-1) (2020 – task 04)

The graph below shows function f .



Therefore,

- A. $f(1) - 2 = f(0)$
- B. $f(0) - 2 = f(2)$
- C. $f(0) - 2 = f(-2)$
- D. $f(1) - 2 = f(-1)$

Solution 3.21. C

Task 3.22. (0-1) (2020 – task 10)

The function f is given by the formula $f(x) = \left(\frac{9}{4}\right)^x$ for each real number x .

For $x = -\frac{3}{2}$ the function f assumes the value of:

- A. $\frac{27}{8}$
- B. $\frac{4}{9}$
- C. $\frac{8}{27}$
- D. $\frac{9}{4}$

Solution 3.22. C

Task 3.23. (0-3) (2020 – task 18)

The quadratic function f is given by the formula $f(x) = -2(x + 1)(x - 3)$.

Complete the following sentences.

- a) The axis of symmetry of the graph of the function f is a line given by the equation
.....
- b) The least value of the function f in the interval $\langle -1, 2 \rangle$ equals
.....
- c) The area of a triangle whose vertices are the points of intersection of the graph of the function f with the axes of the coordinate system equal
.....

Solution 3.23. (a) $x = 1$ (b) 0 (c) 12

Task 3.24. (0-1) (2021 – task 05)

The graph of the function $f(x) = (x + 6)(2x - 4)$ is a parabola whose vertex is a point with coordinates

- A. $(-6,4)$ B. $(6, -4)$ C. $(-6,2)$ D. $(-2, -32)$

Solution 3.24. D

Information for tasks 3.25 – 3.26

A function f assigns to each two-digit number x the remainder of the division of x by 7.

Task 3.25. (0-1) (2021 – task 06)

The set of values of the function f consists of

- A. 10 elements B. 90 elements C. 7 elements D. 13 elements

Solution 3.25. C

Task 3.26. (0-1) (2021 – task 07)

The number of zeros of the function f is equal to

- A. 10 elements B. 90 elements C. 7 elements D. 13 elements

Solution 3.26. D

Task 3.27. (0-1) (2021 – task 08)

The number of positive integers which belong to the set of values of the function

$g(x) = -x^2 - 4x + 21$ is

- A. 10 B. 3 C. 25 D. 2

Solution 3.27. D

$$-x^2 - 4x + 21 > 0 \quad x^2 + 4x - 21 < 0$$

$$\Delta = 16 + 84 = 100 \quad x_1 = \frac{-4-10}{2} = -7 \quad x_2 = \frac{-4+10}{2} = 3$$

$$x \in (-7, 3) \quad (-7, 3) \cap \mathbb{Z}_+ = \{1, 2\}$$

3. Functions

Answers

3.01. C	3.02. B	3.03. A	3.04. D	3.05. C
	3.07. A	3.08. B		
3.11. B		3.13. B	3.14. C	3.15. D
3.16. B	3.17. A		3.19 D	3.20 C
3.21. C	3.22. C		3.24. D	3.25. C
3.26. D	3.27 D			

3.06. (a) $4, -1$ (b) -7 (c) $(-\infty, -3) \cup (1, \infty)$

3.09. (a) 15 (b) 26

3.10

- (a) $\langle -7, 7 \rangle$ (the closed interval)
- (b) $\langle -7, 7 \rangle$ (the interval left-closed and right-open)
- (c) $(5, 7 \rangle$ (the interval left-open and right-closed)
- (d) -7
- (e) $x \in (-5, 5)$

3.12.

- (a) $(-7, 7 \rangle$ (the interval left-open and right-closed)
- (b) $\langle -7, 7 \rangle$ (the closed interval)
- (c) 7
- (d) $\langle -5, 0 \rangle$ (the interval left-closed and right-open)
- (e) 3

3.18 (a) $(-\infty, 2 \rangle$ (b) $\langle -18, \infty$ (c) $(-\infty, -1 \rangle \cup \langle 5, \infty$

3.23 (a) $x = 1$ (b) 0 (c) 12