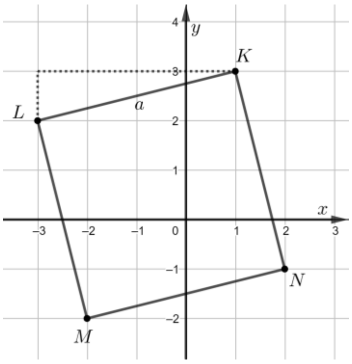
Task 5.01. (0-1) (2015– task 06)

The points $K = (1, 3)$, $L = (-3, 2)$, $M = (-2, -2)$, $N = (2, -1)$ are the vertices of a square. The area of the square is

A. $\sqrt{17}$ **B.** 17 **C.** $\sqrt{34}$ D. 34

Solution 5.01. B



Let a be the side length of the square.

The area of the square is equal to a^2.

By Pythagoras's theorem

$a^2 = 1^2 + 4^2$

$a^2 = 17$

So the area of the square is 17.

Task 5.02. (0-1) (2015 – task 08)

The line k passes through the point $S = (-3,1)$ and is perpendicular to the line l with the equation $y = -\frac{1}{3}x + 12$. The line k has the following equation:

A. $y = -\frac{1}{3}x$ **B.** $y = 3x$ **C.** $y = -\frac{1}{3}x - \frac{8}{3}$ **D.** $y = 3x + 10$

Solution 5.02. D

Let $y = ax + b$ be the slope-interception equation of the line k.

The lines k and l are perpendicular so the product of their gradients is equal to -1.

$$a \times \left(-\frac{1}{3}\right) = -1$$

$$a = 3$$

Now, we know that line k has the equation $y = 3x + b$.

We also know that the line k goes through the point $S = (-3,1)$, which means that the pair of numbers $x = -3$, $y = 1$ satisfies the equation $y = 3x + b$.

$$1 = 3 \times (-3) + b$$

Therefore $b = 10$ and the line k has the equation $y = 3x + 10$.

Task 5.03. (0-1) (2016 – task 11)

The straight lines $2x + 3y - 11 = 0$ and $y = ax$ are perpendicular when

A. $a = -\frac{1}{2}$ **B.** $a = \frac{2}{3}$ **C.** $a = \frac{3}{2}$ **D.** $a = 2$

Solution 5.03. C

Let's convert the general equation $2x + 3y - 11 = 0$ to the slope-intercept equation form:

$2x + 3y - 11 = 0$ $/-2x + 11$

$3y = -2x + 11$ $/\div 3$

$y = -\frac{2}{3}x + \frac{11}{3}$

The lines given by equations $y = -\frac{2}{3}x + \frac{11}{3}$ and $y = ax$ are perpendicular so the product of their gradients is equal to -1:

$-\frac{2}{3} \times a = -1$ $/\times\left(-\frac{3}{2}\right)$

$a = \frac{3}{2}$

Task 5.04. (0-1) (2017 – task 10)

Line k with the equation $y = -\frac{1}{3}x + 11$ is parallel to the line l which contains $K = (-3, 9)$. The equation of the line l is:

A. $y = -\frac{1}{3}x + 10$ **B.** $y = -\frac{1}{3}x + 8$ **C.** $y = 3x + 18$ **D.** $y = 3x$

Solution 5.04. B

Let $y = ax + b$ be the slope-interception equation of the line l.

The lines k and l are parallel so their gradients are equal and thus $a = -\frac{1}{3}$

Now, we know that line l has the equation $y = -\frac{1}{3}x + b$.

We also know that the line l contains the point $K = (-3, 9)$, which means that the pair of numbers $x = -3$, $y = 9$ satisfies the equation $y = -\frac{1}{3}x + b$.

$$9 = -\frac{1}{3} \times (-3) + b$$

Therefore $b = 8$ and the line k has the equation $y = -\frac{1}{3}x + 8$.

Task 5.05. (0-3) (2017 – task 19)

Two points, $M = (-5, -3)$ and $N = (3, 11)$, are located on the Cartesian plane. Complete the following sentences.

(a) The equation of the line MN is ………………………………………… .

(b) The distance of point M from point N is ……………………… .

(c) The midpoint of the segment MN is $S = (x_S, y_S)$, where x_S = ...
and y_S = ...

Solution 5.05. (a) $\boldsymbol{y = \frac{7}{4}x + \frac{23}{4}}$ (b) $\boldsymbol{2\sqrt{65}}$ (c) $\boldsymbol{-1}$ and $\boldsymbol{4}$

(a) Let $y = ax + b$ be the slope-intercept equation of the line MN.

$M(-5, -3) \in MN$, thus $-3 = a \times (-5) + b$.

$N(3, 11) \in MN$, thus $11 = a \times 3 + b$.

This way we have got the set if equations to solve:

$$\begin{cases} -5a + b = -3 \\ 3a + b = 11 \end{cases} \qquad \begin{cases} b = 5a - 3 \\ 3a + (5a - 3) = 11 \end{cases} \qquad \begin{cases} b = \frac{23}{4} \\ a = \frac{7}{4} \end{cases}$$

So the equation of the line MN in slope-intercept form is $y = \frac{7}{4}x + \frac{23}{4}$.

We can also convert this slope-intercept equation to a general form with integer coefficients:

$$y = \frac{7}{4}x + \frac{23}{4}$$

$$\frac{7}{4}x - y + \frac{23}{4} = 0$$

So the line MN in one of its general forms is $7x - 4y + 23 = 0$.

(b) The distance between points M and N is

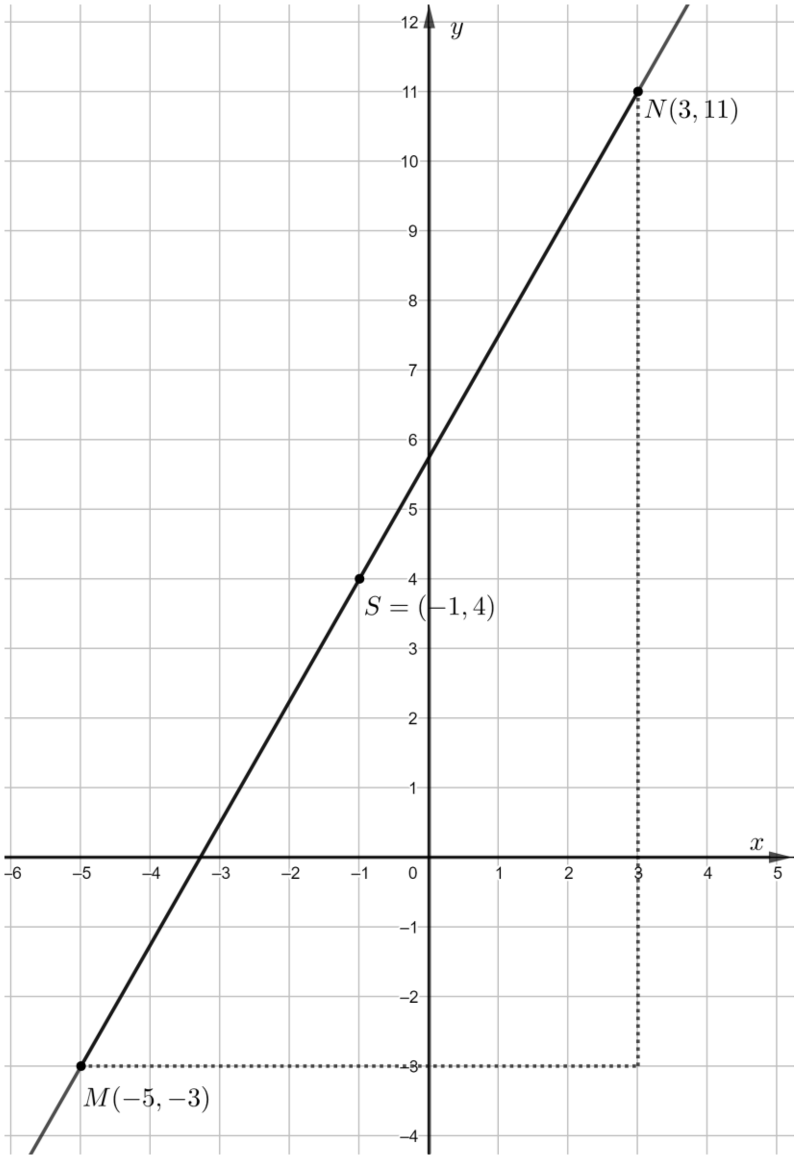
$$|MN| = \sqrt{(3 - (-5))^2 + (11 - (-3))^2}$$

$$|MN| = \sqrt{8^2 + 14^2} = \sqrt{64 + 196} = \sqrt{260} = 2\sqrt{65}$$

(c) The midpoint $S = (x_S, y_S)$ of the segment MN has coordinates:

$$x_S = \frac{-5+3}{2} = -1$$

$$y_S = \frac{-3 + 11}{2} = 4$$



Task 5.05

Information for task 5.6 – 5.8:

Given are points $A(-2,1)$ and $B(3,4)$.

Task 5.06. (0-1) (2018 – task 01)

The length of the line segment AB is equal to

A. $\sqrt{34}$ **B.** $\sqrt{50}$ **C.** $\sqrt{10}$ **D.** $\sqrt{26}$

Solution 5.06. A

$|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$|AB| = \sqrt{(3-(-2))^2 + (4-1)^2}$

$|AB| = \sqrt{5^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34}$

Task 5.07. (0-1) (2018 – task 02)

Points A and B lie on the line

A. $y = \frac{3}{5}x + \frac{1}{5}$ **B.** $y = \frac{3}{5}x + \frac{11}{5}$ **C.** $y = \frac{5}{3}x + \frac{11}{3}$ **D.** $y = \frac{5}{3}x + \frac{7}{3}$

Solution 5.07. B

Let $y = ax + b$ be the slope-intercept equation of the line AB.

$A(-2, 1) \in AB$, thus $1 = a \times (-2) + b$.

$B(3,\ 4) \in AB$, thus $4 = a \times 3 + b$.

This way we have got the set if equations to solve:

$$\begin{cases} -2a + b = 1 \\ 3a + b = 4 \end{cases} \qquad \begin{cases} b = 2a + 1 \\ 3a + (2a + 1) = 4 \end{cases} \qquad \begin{cases} b = \frac{11}{5} \\ a = \frac{3}{5} \end{cases}$$

So the equation of the line AB in slope-intercept form is $y = \frac{3}{5}x + \frac{11}{5}$.

Task 5.08. (0-1) (2018 – task 03)

The centre of the line segment AB is the point

A. $S = \left(\frac{1}{2}, \frac{3}{2}\right)$ **B.** $S = \left(\frac{5}{2}, \frac{3}{2}\right)$ **C.** $S = \left(\frac{5}{2}, \frac{1}{2}\right)$ **D.** $S = \left(\frac{1}{2}, \frac{5}{2}\right)$

Solution 5.08. D

$$S = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = \left(\frac{-2+3}{2}, \frac{1+4}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right)$$

Task 5.09. (0-1) (2018 – task 04)

The line l passes through the point $A = (-5,6)$ and is parallel to the line k given by the equation $y = 2x - 7$. The line l has the following equation:

A. $y = -\frac{1}{2}x + \frac{7}{2}$ **B.** $y = -\frac{1}{2}x + \frac{17}{2}$ **C.** $y = 2x - 4$ **D.** $y = 2x + 16$

Solution 5.09. D

Let $y = ax + b$ be the slope-interception equation of the line l.

The lines k and l are parallel so their gradients are equal and thus $a = 2$

Now, we know that line l has the equation $y = 2x + b$.

We also know that the line l passes through the point $A = (-5,6)$, which means that the pair of numbers $x = -5$, $y = 6$ satisfies the equation $y = 2x + b$:

$$6 = 2 \times (-5) + b$$

Therefore $b = 16$ and the line l has the equation $y = 2x + 16$.

Task 5.10. (0-1) (2019 – task 07)

The line m passes through the point $K = (-2,19)$ and is perpendicular to the line l given by the equation $y = \frac{1}{8}x + 2019$. The equation of the line m is:

A. $y = -8x + 3$

B. $y = -8x + 150$

C. $y = -\frac{1}{8}x + \frac{75}{4}$

D. $y = -\frac{1}{8}x + \frac{3}{8}$

Solution 5.10. A

Let $y = ax + b$ be the slope-interception equation of the line m.

The lines m and l are perpendicular so the product of their gradients is equal to -1.

$$a \times \frac{1}{8} = -1$$

$$a = -8$$

Now, we know that line m has the equation $y = -8x + b$.

We also know that the line m passes through the point $K = (-2, 19)$, which means that the pair of numbers $x = -2$, $y = 19$ fulfils the equation $y = 3x + b$.

$$19 = -8 \times (-2) + b$$

Therefore $b = 3$ and the line k has the equation $y = -8x + 3$.

Task 5.11. (0-4) (2019 – task 18)

Points $A = (8, -1)$, $B = (-4, -23)$, and $K = (1, 9)$ are located on a Cartesian plane. The point S is the midpoint of the line segment AB. The line m is parallel to the line AB and passes through the point K.

Complete the following sentences. Enter the correct numbers in sentences (a) and (b), and write the equation of the line in sentences (c) and (d).

(a) The first coordinate of the point S equals ……………, and the second coordinate of this point is …………… .

(b) The distance between points A and B equals …………… .

(c) The line AB has the equation ………………………… .

(d) The line m has the equation ………………………… .

Solution 5.11. (a) $\mathbf{(2, -12)}$ (b) $\mathbf{2\sqrt{157}}$ (c) $\boldsymbol{y = \frac{11}{6}x - \frac{94}{6}}$ (d) $\boldsymbol{y = \frac{11}{6}x + \frac{43}{6}}$.

(a) $S = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right) = \left(\frac{8 + (-4)}{2}, \frac{-1 + (-23)}{2}\right) = (2, -12)$

(b) $|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$$|AB| = \sqrt{(-4 - 8)^2 + \left(-23 - (-1)\right)^2}$$

$$|AB| = \sqrt{(-12)^2 + (-22)^2} = \sqrt{144 + 484} = \sqrt{628} = 2\sqrt{157}$$

(c) Let $y = ax + b$ be the slope-intercept equation of the line AB.

$A(8, -1) \in AB$, thus $-1 = a \times 8 + b$.

$B(-4, -23) \in AB$, thus $-23 = a \times (-4) + b$.

This way we have got the set if equations to solve:

$$\begin{cases} 8a + b = -1 \\ -4a + b = -23 \end{cases} \qquad \begin{cases} b = -8a - 1 \\ -4a + (-8a - 1) = -23 \end{cases} \qquad \begin{cases} b = -\frac{94}{6} \\ a = \frac{11}{6} \end{cases}$$

So the equation of the line AB in slope-intercept form is $y = \frac{11}{6}x - \frac{94}{6}$.

You can also convert the equation to a general form with integer coefficients, for example:

is $y = \frac{11}{6}x - \frac{94}{6}$ $\quad /\times 6$

$6y = 11x - 94 \qquad /-11x + 94$

$-11x + 6y + 94 = 0$

(d) Let $y = ax + b$ be the slope-interception equation of the line m.

The lines m and AB are parallel, so their gradients are equal and thus $a = \frac{11}{6}$.

Now, we know that line l has the equation $y = \frac{11}{6}x + b$.

We also know that the line l passes through the point $K = (1, 9)$, which means that the pair of numbers $x = 1$, $y = 9$ satisfies the equation $y = \frac{11}{6}x + b$.

$9 = \frac{11}{6} \times 1 + b$

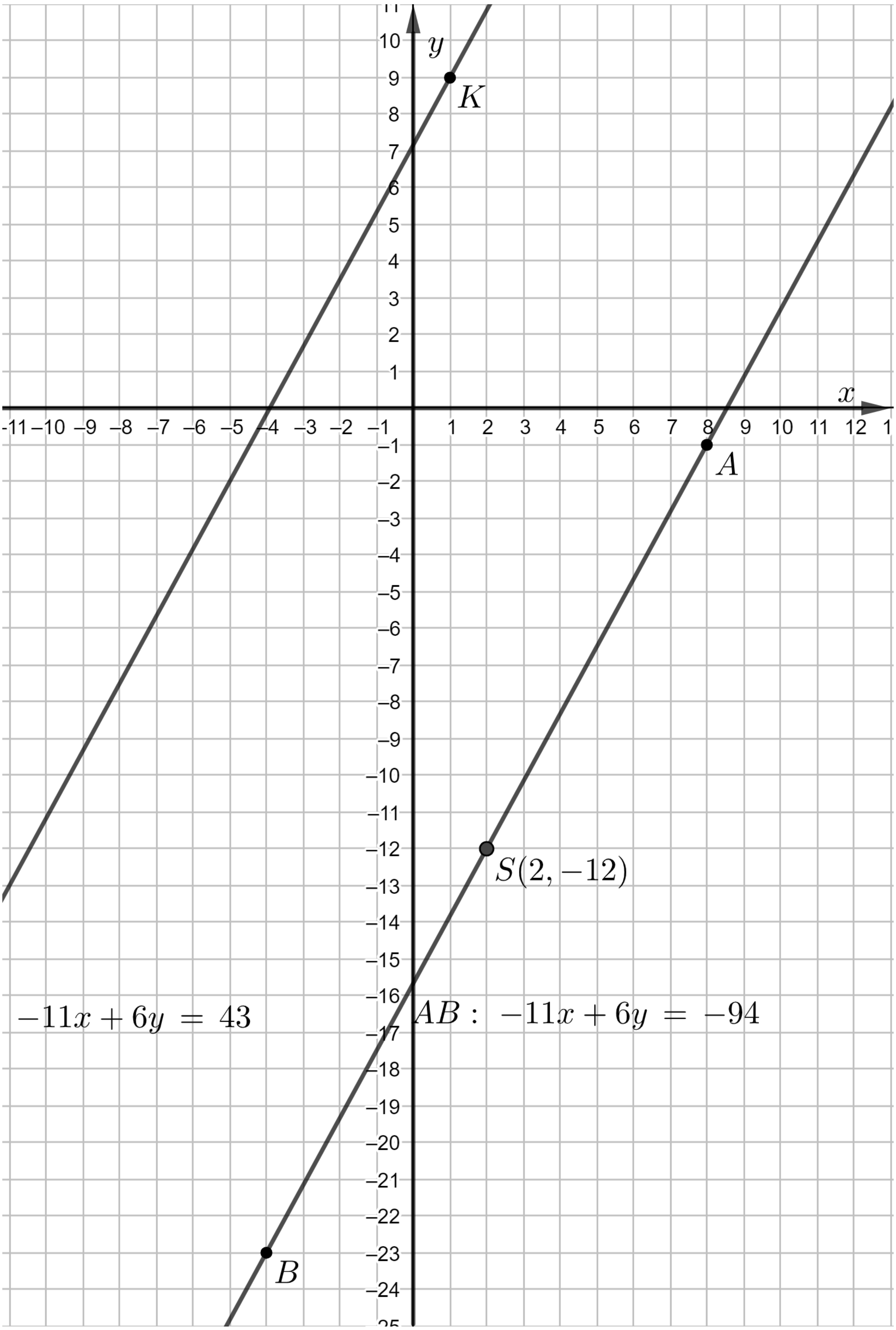
Therefore $b = \frac{43}{6}$ and the line l has the equation $y = \frac{11}{6}x + \frac{43}{6}$.

Again, you can also convert the equation to a general form with integer coefficients, for example:

$y = \frac{11}{6}x + \frac{43}{6} \qquad /\times 6$

$6y = 11x + 43 \qquad /-11x - 43$

$-11x + 6y - 43 = 0$



5.11

Task 5.12. (0-3) (2020 – task 20)

Point $A = (-1, 2)$ is the end point of a line segment *AB*, whereas point $S = \left(1, \frac{1}{2}\right)$ is the midpoint of the line segment *AB* . Complete the following sentences.

(a) The coordinates of point *B* are: …………………………… .

(b) The line segment *AB* is reflected in x-axis. The coordinates of the endpoints of the image of *AB* after reflection are: *A*' = (……. , …….), *B*' = (……. , …….).

(c) The length of the line segment *AB* is ……..……………………………………. .

Solution 5.12. (a) $(\mathbf{3}, \mathbf{-1})$ (b) $(\mathbf{-1}, \mathbf{-2}), (\mathbf{3}, \mathbf{1})$ (c) 5

(a) Let $B = (x_B, y_B)$.

Then $\frac{-1+x_B}{2} = 1$ and $\frac{2+y_B}{2} = \frac{1}{2}$. Thus $x_B = 3$ and $y_B = -1$.

$B = (3, -1)$.

(b) $A' = (x_A, -y_A) = (-1, -2)$ $B' = (x_B, -y_B) = (3, 1)$

(c) $|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$|AB| = \sqrt{(-1-3)^2 + \left(2-(-1)\right)^2}$

$|AB| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$

Task 5.13. (0-5) (2021 – task 17)

The triangle ABC is a right-angled triangle. The length of the hypotenuse AC is equal to 65. The vertex A has coordinates $(-15, 20)$, the vertex B is at the origin of the coordinate system, and the vertex C has both coordinates positive.

Complete the sentences a–c below by writing the correct numeric values in the blanks.

(a) The length of the shortest side of the triangle ABC is equal to ....

(b) The area of the triangle ABC is equal to ....

(c) The radius of the circle circumscribed on the triangle ABC is equal to ....

5. Analytic geometry

Write the equation of the straight line $\boldsymbol{BC}$.

(d) The straight line BC has the equation ....

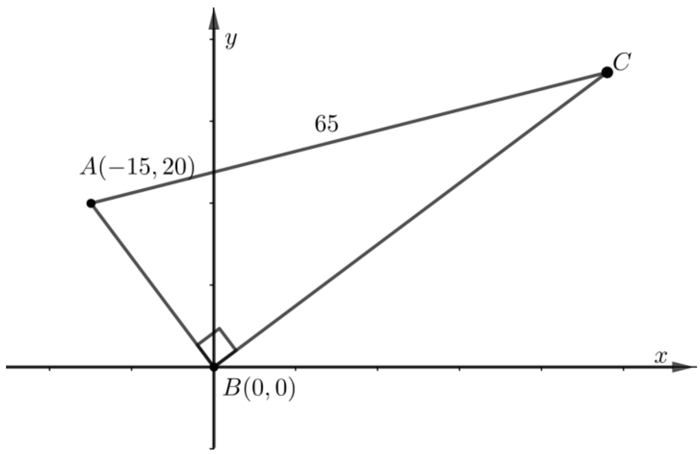
Write the coordinates of the centre of the circle circumscribed on the triangle $\boldsymbol{ABC}$.

(e) The centre of the circle circumscribed on the triangle ABC has coordinates

....

Solution 5.13 (a) 25 (b) 750 (c) 32.5 (d) $\boldsymbol{y = \frac{3}{4}x}$ (e) $\boldsymbol{\left(16\frac{1}{2}, 28\right)}$

(a)



- $|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

  $|AB| = \sqrt{\left(0-(-15)\right)^2 + (0-20)^2} = \sqrt{225+400} = \sqrt{625}$

  $|AB| = 25$
- $|BC|^2 + |BA|^2 = |AC|^2$

  $|BC|^2 + 25^2 = 65^2$

  $|BC|^2 = 65^2 - 25^2$

  $|BC| = 60$
- The lengths of triangle of sides are $|AB| = 25$, $|BC| = 60$, $|AC| = 65$.
- So the length of the shortest side is 25.

(b) The area of the triangle ABC is equal $\frac{|AB|\times|BC|}{2} = \frac{25\times 60}{2} = 750$

(c) The triangle ABC is a right angled triangle so the radius of the circle circumscribed on it is the half of its hypotenuse: $R = \frac{65}{2} = 32.5$

(d) First we are going to find out the equation of the line AB and then BC.

1. Let $y = ax + b$ be the slope-intercept equation of the line AB.

   $A(-15, 20) \in AB$, thus $20 = a \times (-15) + b$.

   $B(0,\ 0) \in AB$, thus $0 = a \times 0 + b$.

   This way we have got the set if equations to solve:

   $$\begin{cases} 0a + b = 0 \\ -15a + b = 20 \end{cases} \qquad \begin{cases} b = 0 \\ -15a + 0 = 20 \end{cases} \qquad \begin{cases} b = 0 \\ a = -\frac{4}{3} \end{cases}$$

   So the equation of the line AB in slope-intercept form is $y = -\frac{4}{3}x$.

2. Let $y = cx + d$ be the slope-intercept equation of the line BC.

   $B(0,\ 0) \in BC$, thus $0 = c \times 0 + d$ and so $d = 0$.

   We know that the triangle ABC is a right angled triangle and BC is its hypotenuse. For this reason the lines AB and BC are perpendicular and therefore the product of their gradients is equal to -1:

   $$a \times c = -1 \qquad -\frac{4}{3} \times c = -1 \qquad c = \frac{3}{4}$$

   So the equation of the line BC in slope-intercept form is $y = \frac{3}{4}x$.

(e) First we will find coordinates of the point C and then the centre O, which is the middle of the segment AC.

1. The point $C(x_C, y_C)$ is on line BC, so $y_C = \frac{3}{4}x_C$.

   Moreover $|AC| = 65$ so

   $$\sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = 65$$

   $$\sqrt{(x_C - (-15))^2 + \left(\frac{3}{4}x_C - 20\right)^2} = 65$$

   $$\sqrt{(x_C + 15)^2 + \left(\frac{3}{4}x_C - 20\right)^2} = 65$$

   $$(x_C + 15)^2 + \left(\frac{3}{4}x_C - 20\right)^2 = 65^2$$

   $$x_C{}^2 + 30x_C + 225 + \frac{9}{16}x_C{}^2 - 30x_C + 400 = 4225$$

   $$\frac{25}{16}x_C{}^2 = 3600$$

$$x_C{}^2 = \frac{3600 \times 16}{25}$$

$$x_C{}^2 = 36 \times 4 \times 16$$

From the assumption, the coordinates of point C are positive, so

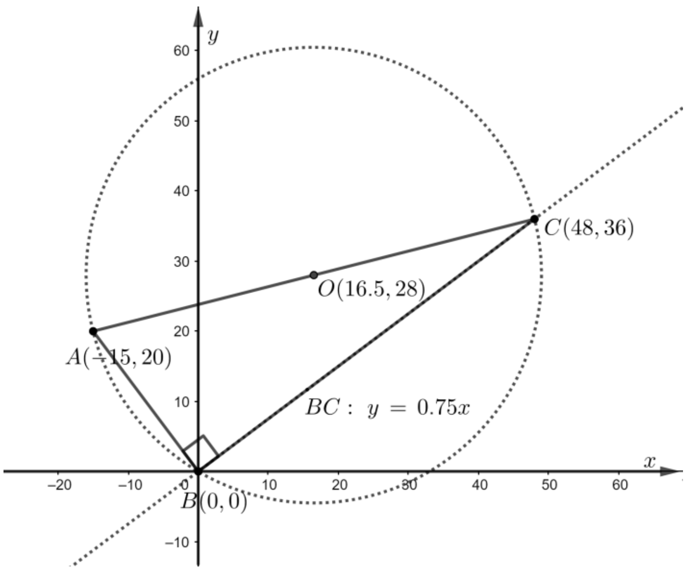
$$x_C = 6 \times 2 \times 4$$

$$x_C = 48$$

$$y_C = \frac{3}{4} \times 48$$

$$y_C = 36$$

2. The centre $\mathrm{O}(x_o, y_O) = \left(\frac{x_A + x_C}{2}, \frac{y_A + y_C}{2}\right) = \left(\frac{-15+48}{2}, \frac{20+36}{2}\right) = \left(16\frac{1}{2}, 28\right)$



Answers

5.01. B

5.02. D

5.03. C

5.04. B

5.05. B

5.06. B

5.07. B

5.08. D

5.09. D

5.10. A

5.11. (a) $(2, -12)$ (b) $2\sqrt{157}$ (c) $y = \frac{11}{6}x - \frac{94}{6}$ (d) $y = \frac{11}{6}x + \frac{43}{6}$

5.12. (a) $B = (3, -1)$ (b) $A' = (-1, -2)$, $B' = (3, 1)$ (c) $|AB| = 5$

5.13 (a) 25 (b) 750 (c) 32.5 (d) $y = \frac{3}{4}x$ (e) $\left(16\frac{1}{2}, 28\right)$