Information for tasks 6.1 – 6.2.

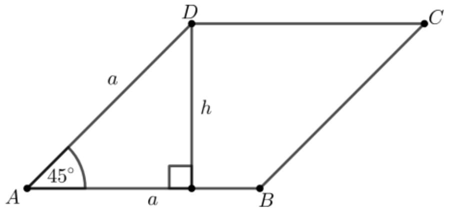
The acute angle of a rhombus is $45°$ and the area of the rhombus is $100\sqrt{2}$.

Task 6.01. (0-1) (2015 – task 09)

The height of the rhombus is

A. $20\sqrt{2}$ **B.** 20 **C.** $10\sqrt{2}$ D. 10

Solution 6.01 D



Let a be the length of the side of the rhombus.

Let h be the height of the rhombus.

The area of the rhombus is $100\sqrt{2}$, and the acute angle between sides is $45°$, so

$$a \times a \times \sin(45°) = 100\sqrt{2}$$

$$a^2 \times \frac{\sqrt{2}}{2} = 100\sqrt{2}$$

$$a = 10\sqrt{2}$$

$$\frac{h}{a} = \sin(45°)$$

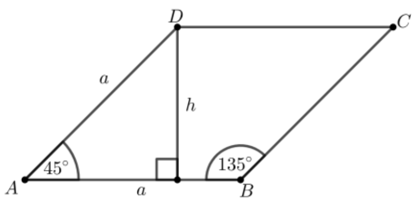
$$h = a\sin(45°) = 10\sqrt{2} \times \frac{\sqrt{2}}{2} = 10$$

Task 6.02. (0-1) (2015 – task 10)

The tangent of the obtuse angle of the rhombus is equal to

A. -1 **B.** 1 **C.** $\frac{\sqrt{2}}{2}$ **D.** $-\frac{\sqrt{2}}{2}$

Solution 6.02 A

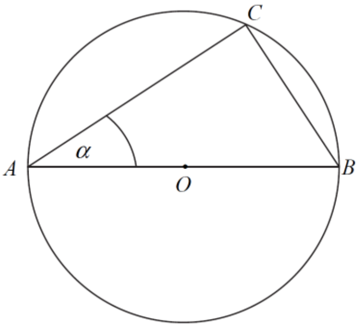


Acute and obtuse angles add up to $180°$ in a rhombus.

So if the acute angle measures $45°$ then the measure of the obtuse angle is $135°$ and we can calculate its tangent using the tangent formula $\tan(135°) = \tan(180° - 45°) = -\tan(45°) = -1$.

Task 6.03. (0-1)   (2015 – task 10)

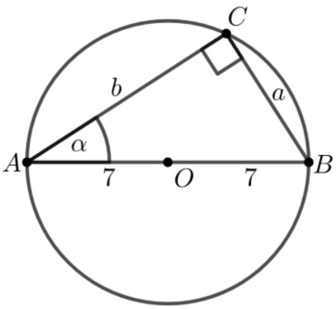
The triangle ABC is circumscribed by a circle with a radius of 7 cm. (see illustration). The centre O of a circle lies on the side AB, and the cosine of the angle BAC is equal to $\frac{2\sqrt{10}}{7}$.



The length of the line segment BC is equal to

A. $\frac{\sqrt{10}}{49}$ **B.** 6   **C.** $\frac{2\sqrt{10}}{49}$ D. 3

Solution 6.03 B



$\cos\alpha = \frac{b}{14}$

$\frac{2\sqrt{10}}{7} = \frac{b}{14}$

$b = 4\sqrt{10}$

$a^2 = 14^2 - \left(4\sqrt{10}\right)^2$

$a^2 = 196 - 160$

$a = 6$

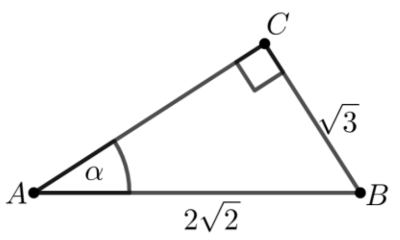
Task 6.04. (0-1) (2016 – task 07)

In a right-angled triangle, one of the shorter sides is $\sqrt{3}$ long, and the angle opposite that side is α . The length of the hypotenuse of this triangle is $2\sqrt{2}$.

The value of the expression $\frac{1}{\sin\alpha}$ is

A. $\frac{\sqrt{11}}{8}$ **B.** $\frac{2\sqrt{2}}{3}$ **C.** $\frac{2\sqrt{6}}{3}$ **D.** $\frac{\sqrt{22}}{4}$

Solution 6.04 C



$$\sin\alpha = \frac{\sqrt{3}}{2\sqrt{2}}$$

$$\frac{1}{\sin\alpha} = \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{6}}{3}$$

Task 6.05. (0-1) (2016 – task 08)

The points K, L, and M are colinear, and point M is located between points K and L. It is also known that $|KL| = 11$ and $|LM| = 5|KM|$. In that case, the length of the line segment LM is

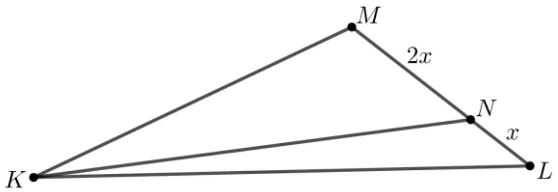
A. $\frac{55}{6}$ **B.** $\frac{11}{6}$ **C.** $\frac{11}{5}$ **D.** $\frac{33}{5}$

Solution 6.05 A

Let x be the length of the KM segment. Then, based on the assumption, the LM segment has the length $5x$. Therefore $6x = 11$, and so $x = \frac{11}{6}$ and $|LM| = 5x = \frac{55}{6}$.

Task 6.06. (0-1)   (2016 – task 09)

On the side LM of the triangle KLM the point N was selected so that the length of the line segment MN is twice the length of the line segment LN. The area of the triangle KLN equals 7.5. Thus, the area of the triangle KLM equals

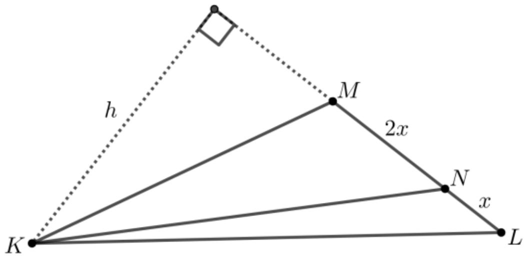


A. 15   **B.** 18.75   **C.** 22.5   D. 30

Solution 6.06 C

Note that the triangles KLN and KLM have the same height h from the vertex K.

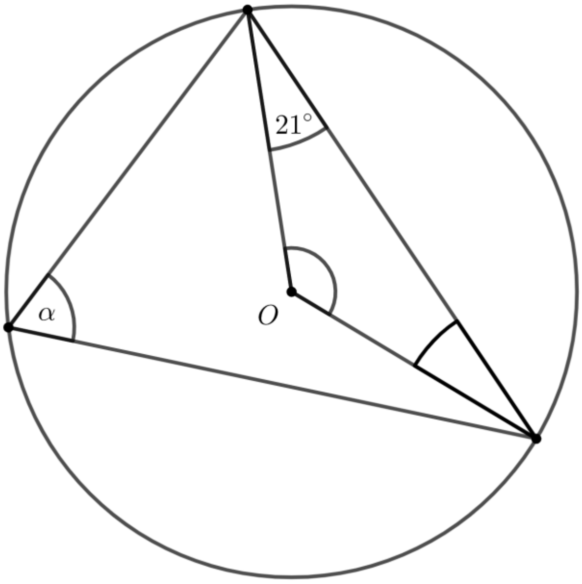
The area of the triangle KLN is $\frac{xh}{2}$ and the area if the triangle KLM is $\frac{3xh}{2}$



which is three times greater than the area of KLN.
We know that the area of KLN is 7.5 so the area of KLM is 3×7.5 and this 22.5.

Task 6.07. (0-1) (2016 – task 10)

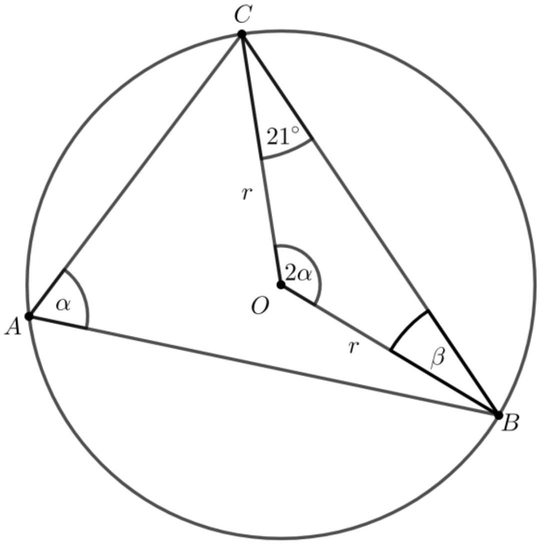
A triangle was inscribed into a circle with the centre point O as shown in the illustration. The angle α of this triangle is



A. 28° **B.** 42° **C.** 63° **D.** 69°

Solution 6.07 D

The triangle COB is an isosceles triangle because OB and OC are radii of the circle and so $\beta = 21°$. The inscribed angle BAC is subtended on the same arc as the central angle BOC so the angle BOC measures 2α .



$$2\alpha + \beta + 21° = 180°$$
$$2\alpha + 21° + 21° = 180°$$
$$2\alpha = 132°$$
$$\alpha = 69°$$

Task 6.08. (0-2) (2016 – task 17)

In an isosceles trapezium, the internal acute angle is 60°. The side and the shorter base are equal. The perimeter of the trapezium equals 50. Complete the following sentences.

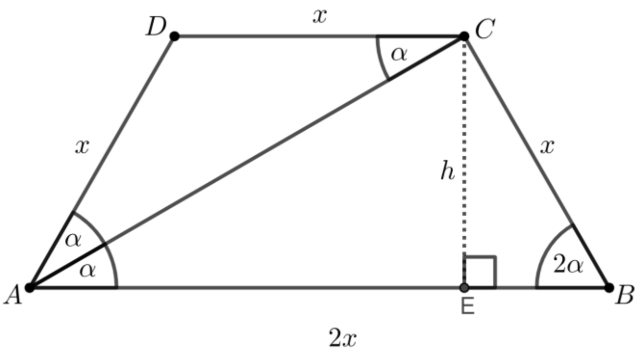
(a) The shorter base of the trapezium equals ………………… .

(b) The area of the trapezium equals ………………… .

Solution 6.08 (a) **10** (b) $\mathbf{75\sqrt{3}}$

Let $\alpha = 30°$ and let x be the length of the shorter base the trapezium.

Using properties of special triangles with angles $30°, 60°, 90°$ we can shorten the reasoning in the picture below.



$5x = 50$

$x = 10$

$\sin(2\alpha) = \frac{h}{x}$

$\sin 60° = \frac{h}{10}$

$h = 10\sin 60°$

$h = 10 \times \frac{\sqrt{3}}{2}$

$h = 5\sqrt{3} \times \frac{\sqrt{3}}{2}$

The area of the trapezium equals $\frac{(2x+x)h}{2} = \frac{3xh}{2} = \frac{3\times10\times5\sqrt{3}}{2} = 75\sqrt{3}$

Task 6.09. (0-1) (2017 – task 08)

α is a positive acute angle. The cosine value of α is three times greater than its sine value. Therefore, the tangent value of α is:

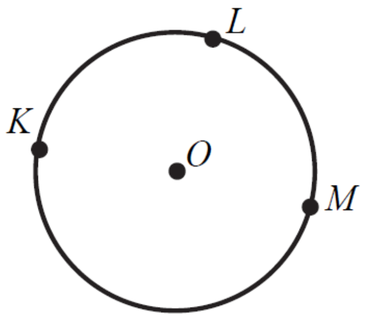
A. $\frac{1}{3}$ **B.** 3 **C.** $\sqrt{10}$ **D.** $\frac{\sqrt{10}}{10}$

Solution 6.09 A

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{3\times\sin\alpha} = \frac{1}{3}$$

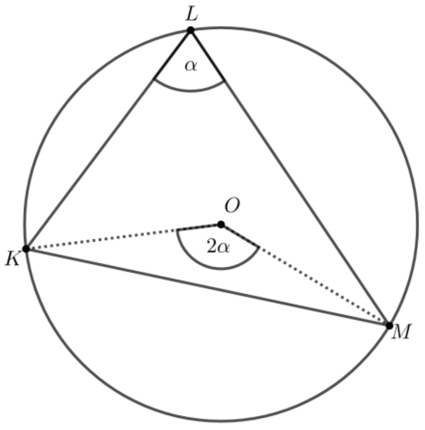
Task 6.10. (0-1) (2017 – task 09)

K, *L* and *M* are three points which lie on a circle with centre *O* (see the illustration). The obtuse angle *KOM* is 170°. The acute angle KLM is:



A. 85° **B.** 80° **C.** 75° **D.** 70°

Solution 6.10 A



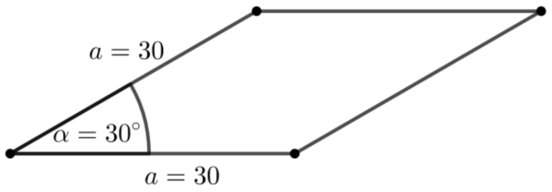
$$2\alpha = 170°$$
$$\alpha = 85°$$

Task 6.11. (0-1)   (2017 – task 10)

The acute angle of a rhombus is 30° and the length of one side of the rhombus is 30. The area of the rhombus is:

A. 900   **B.** 90   **C.** 450   **D**. 45

Solution 6.11 C



$$\text{Area} = a \times a \times \sin\alpha$$

$$\text{Area} = 30 \times 30 \times \sin 30°$$

$$\text{Area} = 900 \times \frac{1}{2} = 450$$

Task 6.12. (0-3)   (2018 – task 14)

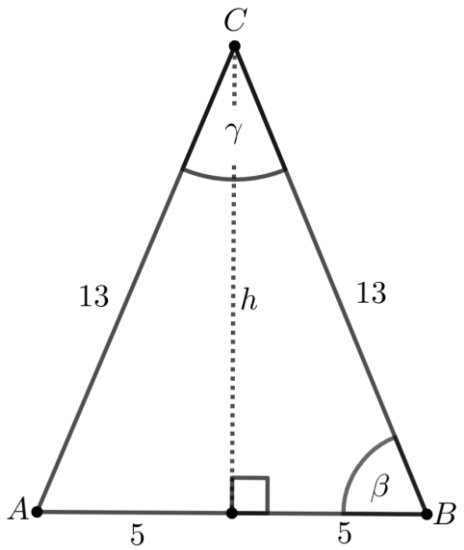
ABC is a triangle with $|AC| = |BC| = 13$ and $|AB| = 10$. Complete the following sentences.

(a) The area of the triangle *ABC* equals …………… .

(b) The sine of the angle *ACB* equals ……………… .

(c) The sine of the angle *ABC* equals ……………… .

Solution 6.12 (a) **60**   (b) $\frac{120}{169}$ (c) $\frac{12}{13}$



$$h^2 = 13^2 - 5^2$$

$$h = 12$$

$$\text{Area}_{ABC} = \frac{10 \times 12}{2} = 60$$

$$\text{Area}_{ABC} = \frac{13 \times 13 \times \sin\gamma}{2} = 60$$

$$\sin\gamma = \frac{120}{169}$$

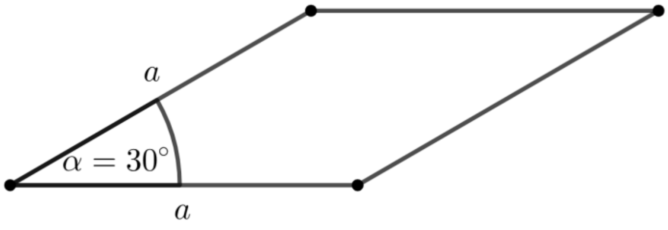
$$\sin\beta = \frac{12}{13}$$

Task 6.13 (0-1) (2019 – task 08)

The acute angle of a rhombus is 30° , and the area of the rhombus is $\frac{361}{2}$. The side length of this rhombus is

A. 76 **B.** $76\sqrt{2}$ **C.** 19 **D.** $19\sqrt{2}$

Solution 6.13 C



$$\text{Area} = a \times a \times \sin 30°$$

$$\text{Area} = a \times a \times \frac{1}{2}$$

$$a \times a \times \frac{1}{2} = \frac{361}{2}$$

$$a^2 = 361$$

$$a = 19$$

Task 6.14. (0-1) (2019 – task 09)

The sine of an obtuse angle α is: $\sin\alpha = \frac{2\sqrt{2}}{3}$. Therefore the cosine of this angle equals

A. $\cos\alpha = \frac{1}{9}$ B. $\cos\alpha = \frac{1}{3}$ C. $\cos\alpha = -\frac{1}{3}$ D. $\cos\alpha = -\frac{1}{9}$

Solution 6.14 C

$$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \left(\frac{2\sqrt{2}}{3}\right)^2 = 1 - \frac{8}{9} = \frac{1}{9}$$

The cosine of an obtuse angle is a negative number, so $\cos\alpha = -\frac{1}{3}$.

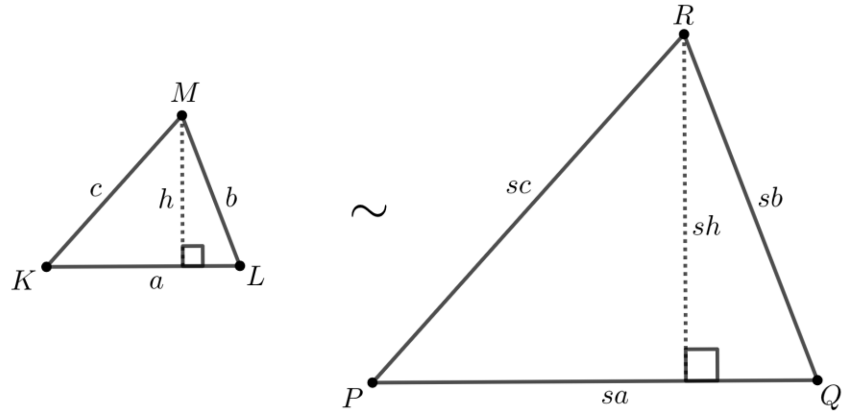
Task 6.15. (0-1)   (2019 – task 13)

Triangles KLM and PQR are similar. The area of the triangle KLM is 6, and the area of the triangle PQR is 90 units greater than the area of the KLM triangle. The perimeter of the triangle KLM equals 12. Hence, the perimeter of the triangle PQR equals:

A. 102   **B.** 48   **C.** 768   **D.** 192

Solution 6.15 B

Let s be a scale factor of similarity KLM to PQR.



The ratio of areas of similar shapes is equal to s^2.

$$s^2 = \frac{\text{Area}(KLM)}{\text{Area}(PQR)} = \frac{6}{6+90} = \frac{6}{96} = \frac{1}{16}$$

$$s = \frac{1}{4}$$

The ratio of perimeters of similar shapes is equal to s.

$$s = \frac{\text{Perimeter}(KLM)}{\text{Perimeter}(PQR)}$$

$$\frac{1}{4} = \frac{12}{\text{Perimeter}(PQR)}$$

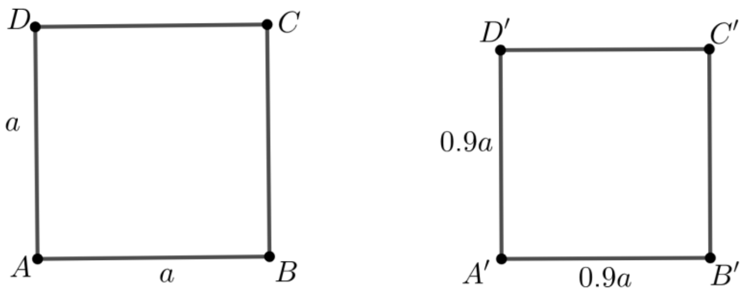
$$\text{Perimeter}(PQR) = 48$$

Task 6.16. (0-1) (2020 – task 07)

The length of the side of a square is reduced by 10 percent. Then, the area of the square will be reduced by:

A. 9% B. 10% C. 19% D. 81%

Solution 6.16 C



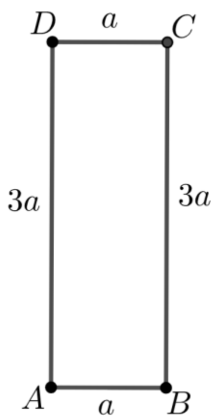
$$\frac{a^2-(0.9a)^2}{a^2}=\frac{a^2-0.81a^2}{a^2}=\frac{0.19a^2}{a^2}=0.19=19\%$$

Task 6.17. (0-1) (2020 – task 11)

The area of a rectangle is 27. One side of this rectangle is 3 times the length of the other side. The perimeter of the rectangle is:

A. 12 B. 18 C. 24 D. 27

Solution 6.17 C



$$\text{Area}(ABCD)=27$$

$$a\times 3a=27$$

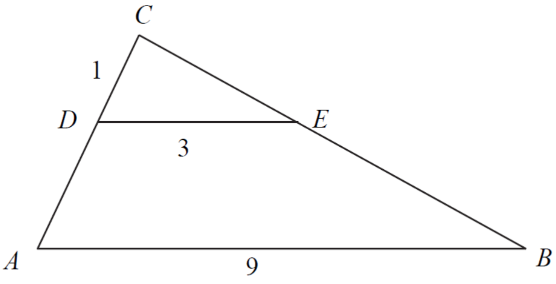
$$a^2=9$$

$$a=3$$

$$\text{Perimeter}(ABCD)=8a=24$$

Task 6.18. (0-1) (2020 – task 14)

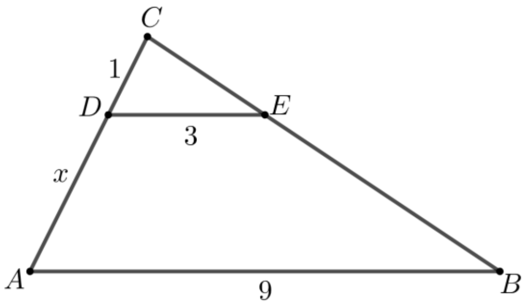
In the triangle ABC, the line segments AB and DE are parallel (refer to the figure below), and $|CD| = 1$, $|DE| = 3$ and $|AB| = 9$.



Hence

A. $|AD| = 2$ **B.** $|AD| = \frac{7}{3}$ **C.** $|AD| = 3$ **D.** $|AD| = \frac{10}{3}$

Solution 6.18 A



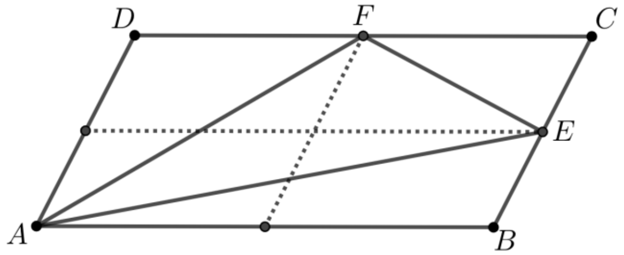
$$\frac{x+1}{1} = \frac{9}{3}$$
$$x + 1 = 3$$
$$x = 2$$

Task 6.19. (0-1) (2021 – task 10)

The area of the parallelogram $ABCD$ is equal to P. Points E and F are the midpoints of the sides BC and CD respectively. The area of the triangle AEF is equal to

A. $\frac{1}{8}P$ **B.** $\frac{1}{4}P$ **C.** $\frac{3}{8}P$ **D.** $\frac{1}{2}P$

Solution 6.19 C



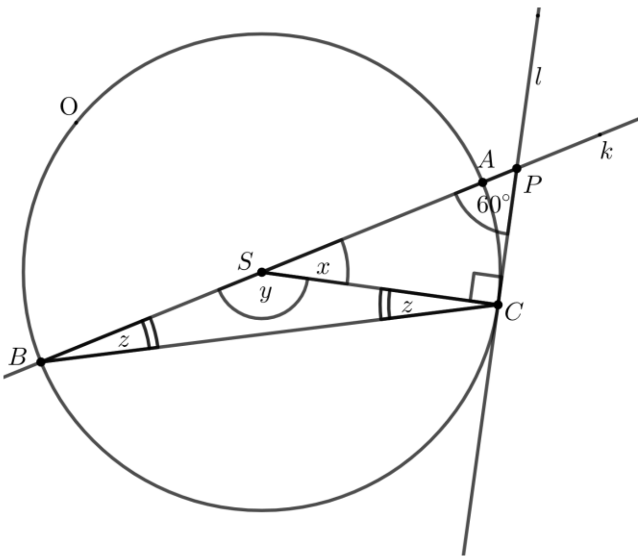
$$\text{Area}(ABCD) = P$$
$$\text{Area}(ABE) = \frac{1}{4}P$$
$$\text{Area}(AFD) = \frac{1}{4}P$$
$$\text{Area}(FEC) = \frac{1}{8}P$$
$$\text{Area}(AEF) = P - \frac{1}{4}P - \frac{1}{4}P - \frac{1}{8}P = \frac{3}{8}P$$

Task 6.20. (0-1) (2021 – task 11)

We are given a circle O and a point P outside the circle. Lines k and l pass through the point P. The line k crosses the circle O at points A and B (where $|PA| < |PB|$) and passes through its centre. The line l is a tangent to the circle O at point C. The angle between the lines k and l is 60°. The angle CBA is equal to

A. 15° **B.** 30° **C.** 45° **D.** 60°

Solution 6.20 A



$x = 180° - 90° - 60° = 30°$

$y = 180° - 30° = 150°$

$2z + 150° = 180°$

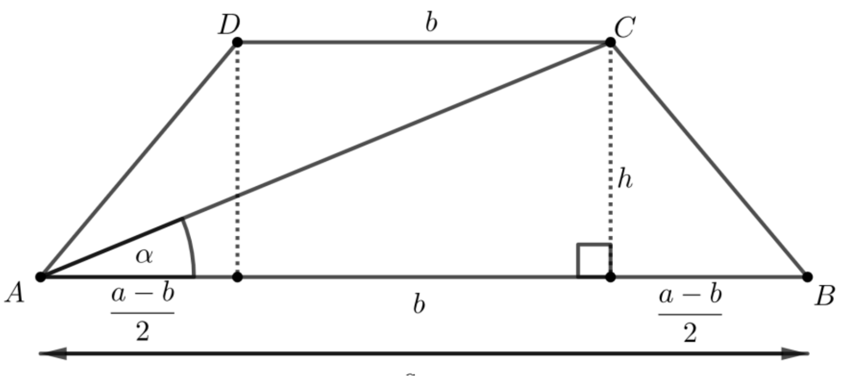
$z = 15°$

Task 6.21. (0-1) (2021 – task 12)

The arithmetic mean of the lengths of the bases of an isosceles trapezium is equal to 9, and the area of the trapezium is equal to 36. The tangent of the angle between the diagonal of the trapezium and the base of the trapezium is equal to

A. $\frac{9}{4}$ **B.** $\frac{4}{9}$ **C.** $\frac{1}{4}$ **D**. $\frac{1}{9}$

Solution 6.21 B



$\text{Area}(ABCD) = 36$

$\frac{a+b}{2} \times h = 36$ $/ \ \frac{a+b}{2} = 9$

$9h = 36$

$h = 4$

$$\tan\alpha = \frac{h}{\frac{a-b}{2} + b} = \frac{h}{\frac{a+b}{2}} = \frac{4}{9}$$

Answers

6.01 D

6.02 A

6.03 B

6.04 C

6.05 A

6.06 C

6.07 (a) 10 (b) $73\sqrt{3}$

6.08 D

6.09 A

6.10 A

6.11 C

6.12 (a) 60 (b) $\frac{120}{169}$ (c) $\frac{12}{13}$

6.13 C

6.14 C

6.15 B

6.16 C

6.17 C

6.18 A

6.19 C

6.20 A

6.21 B