7. Probability and statistics

Task 7.01. (0-2) (2015 – task 13)

From the set of numbers {1, 2, 3, ..., 8}, a single number is randomly drawn two times, without replacement. Complete the following sentences.

a) Event *A* – the product of the two randomly drawn numbers is divisible by 5. This means that one of the randomly drawn numbers must be ....
b) The probability of event *A* is equal to ....

Solution 7.01. (a) 5 (b) $\frac{1}{4}$

The set of elementary events

$\Omega = \{(x, y);\ x\epsilon\{1, 2, 3, \dots, 8\}\ and\ \ y\epsilon\{1, 2, 3, \dots, 8\} - \{x\}\}$

Number of all elementary events

$\overline{\overline{\Omega}} = 8 \times 7 = 56$

The event *A* as a set of elementary events

$\mathrm{A} = \{(x, y);\ (x, y)\epsilon\Omega \text{ and } x \times y \text{ is dividible by } 5\}$

$\mathrm{A} = \{(x, 5);\ x\epsilon\{1, 2, 3, \dots, 8\} - \{5\}\} \cup \{(5, y);\ y\epsilon\{1, 2, 3, \dots, 8\} - \{5\}\}$

The number of elementary events favouring *A*

$\overline{\overline{A}} = 7 + 7 = 14$

Probability of the event *A*

$P(A) = \frac{\overline{\overline{A}}}{\overline{\overline{\Omega}}} = \frac{14}{56} = \frac{1}{4}$

Task 7.02. (0-1) (2016 – task 12)

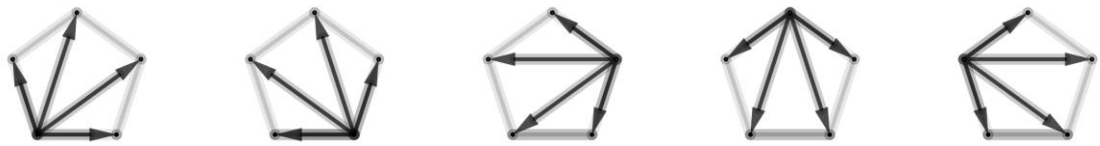
Five different points are located on one plane, and any three of these points are non-colinear. The number of line segments which have their endpoints at any two of these five points is

A. 5 **B.** 10 **C.** 20 **D.** 15

7. Probability and statistics

Solution 7.02. B

If we drew a segment from every point to the other point then we would draw 5×4 segments.

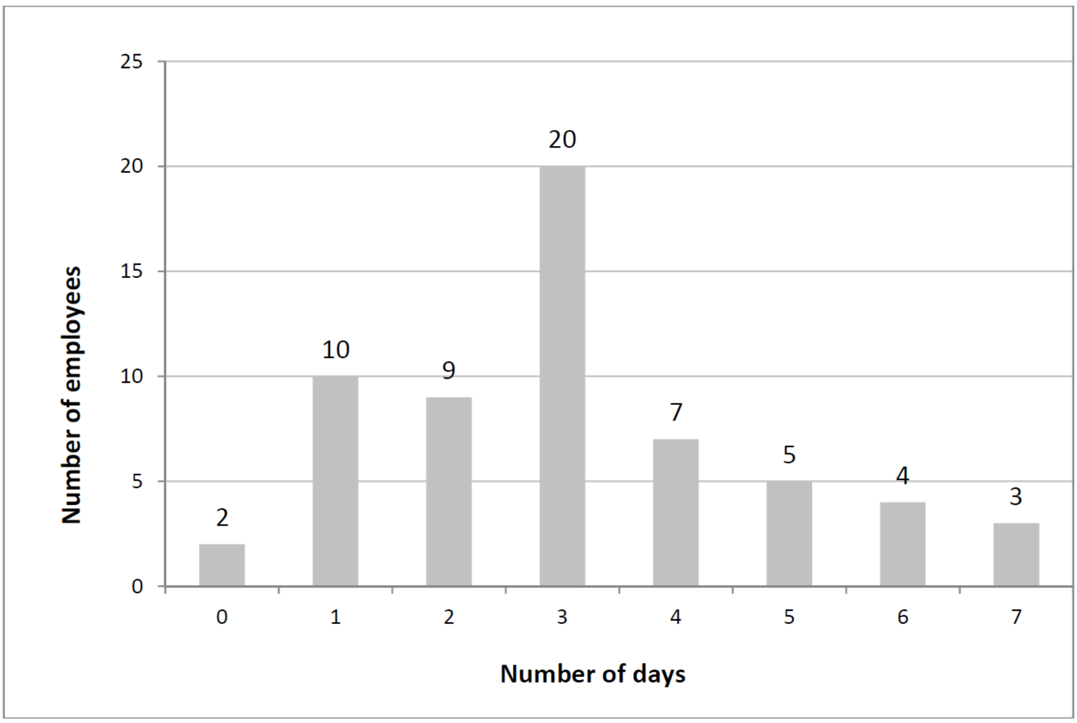


But then every segment would be drawn two times because each segment has two ends.

So the number of all segment is equal to $\frac{5\times 4}{2}$ which is 10.

Task 7.03. (0-2) (2016 – task 14)

Each of the 60 employees of a company was asked to give the number of days on which they went grocery shopping in the previous week. The survey results are presented in the chart below.



7. Probability and statistics

Complete the following sentences using the chart.

(a) The percentage of the employees who shopped for groceries on more than four days in the previous week is ....

(b) The median number of days when the employees shopped for groceries equals ....

Solution 7.03. (a) 20%   (b) 3

(a) $\frac{5+4+3}{60} = \frac{1}{5} = 20\%$ (b) 3

(b) Let's put all data in order from the least to the greatest.

Data	Frequency	Data are located on places between
0	2	1-2
1	10	3-12
2	9	13-21
3	20	22-41
4	7	42-48
5	5	49-53
6	4	54-57
7	3	58-60

The two middle data are located at **thirtieth** and **thirty first places**, which both are equal to 3, so the median is $\frac{3+3}{2} = 3$

Task 7.04. (0-1)   (2017 – task 13)

The set of numbers $\langle 1, 200)$ contains exactly k natural even numbers which are not divisible by 3. Therefore:

A. $k = 67$ **B.** $k = 66$ **C.** $k = 34$ **D.** $k = 33$

7. Probability and statistics

Solution 7.04. B

Even numbers that are not divisible by 3 are of the form

$2(3n+1) = 6n+2$ or $2(3n+2) = 6n+4$

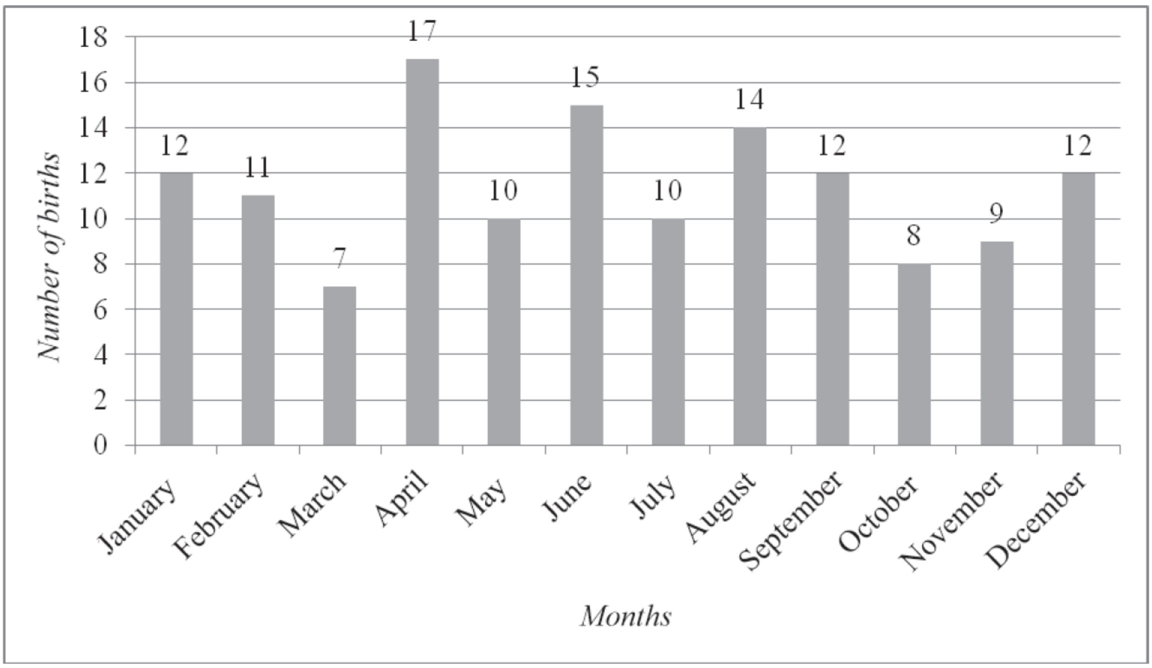
where n is an integer. We are looking for these numbers in the set $\{1, 2, 3, \dots 199\}$.

Numbers of the form $6\boldsymbol{n}+2$	Numbers of the form $6\boldsymbol{n}+4$
$2 = 6 \times \mathbf{0} + 2$	$4 = 6 \times \mathbf{0} + 4$
$8 = 6 \times \mathbf{1} + 2$	$10 = 6 \times \mathbf{1} + 4$
$\dots$	$\dots$
$194 = 6 \times \mathbf{32} + 2$	$196 = 6 \times \mathbf{32} + 4$

As we can see there 33 numbers of each form (like from 0 to 32 including) so in total there are 66 such numbers. So $k = 66$.

Task 7.05. (0-2) (2017 – task 16)

The diagram below shows the number of births in individual months of 2016 in X, an urban-rural municipality.



7. Probability and statistics

Complete the following sentences.

(a) The median of the dataset which contains the monthly numbers of births in 2016 in municipality X equals …………………… .

(b) In 2016, the birth of the hundredth new born baby in municipality X took place in the month of ……………………………………………… .

Solution 7.05. (a) 11.5 (b) September

(a) Unordered data: 12, 11, 7, 17, 10, 15, 10, 14, 12, 8, 9, 12

Ordered data: 7, 8, 9, 10, 10, **11, 12**, 12, 12, 14, 15, 17

Median = $\frac{11+12}{2} = 11.5$

(b) January: 12<100,

February: 12+11=23<100,

March: 12+11+7=30<100,

April: 12+11+7+17=47<100,

May: 12+11+7+17+10=57<100,

June: 12+11+7+17+10+15=72<100,

July: 12+11+7+17+10+15+10=82<100,

August: 12+11+7+17+10+15+10+14=96<100,

September: 12+11+7+17+10+15+10+14+12=108>100,

Task 7.06. (0-2) (2017 – task 18)

A random experiment consists in a simultaneous toss of two coins and a cubic dice. The result of tossing a coin may be heads or tails. Each of the six faces of the dice contains a different number of dots. The number of the dots belongs to the set $\{1, 2, 3, 4, 5, 6\}$. Complete the following sentences.

(a) The probability of an event the result of which is two tails and a face with six dots is …………………… .

(b) The probability of an event whose result is two tails and a face with an even number of dots is …………………… .

Solution 7.06. (a) $\frac{\mathbf{11}}{\mathbf{144}}$ (b) $\frac{\mathbf{3}}{\mathbf{16}}$

The sample space Ω (the set if all possible outcomes in the experiment):

$\Omega = \{(x, y, a, b);\ x, y \in \{T, H\}\ and\ a, b \in \{1, 2, 3, 4, 5, 6\}\ \}$

The cardinality of the set Ω (the number of all possible outcomes):

$\overline{\overline{\Omega}} = 2 \times 2 \times 6 \times 6 = 144$

(a) Let A be the set of outcomes that describes the result of two tails and a face with six dots . Then A is the union of three excluding sets

$A_1 = \{(T, T, a, b);\ \ a \in \{\ 6\}\ \text{and}\ \ b \in \{1, 2, 3, 4, 5\}\ \}$

$A_2 = \{(T, T, a, b);\ \ a \in \{1, 2, 3, 4, 5\}\ \text{and}\ \ b \in \{\ 6\}\ \ \}$

$A_3 = \{(T, T, a, b);\ \ a \in \{6\}\ \text{and}\ \ b \in \{\ 6\}\ \}$

$\overline{\overline{A}} = 5 \times 1 + 1 \times 5 + 1 \times 1 = 11$

Probability of the event A: $P(A) = \frac{\overline{\overline{A}}}{\overline{\overline{\Omega}}} = \frac{11}{144}$

(b) Let B be the set of outcomes that describes the result of two tails and a face with an even number of dots. Then B is the union of three excluding sets

$B_1 = \{(T, T, a, b);\ \ a \in \{\ 2, 4, 6\}\ \text{and}\ \ b \in \{1, 3, 5\}\ \}$

$B_2 = \{(T, T, a, b);\ \ a \in \{1, 3, 5\}\ \ \text{and}\ \ b \in \{\ 2, 4, 6\}\}$

$B_3 = \{(T, T, a, b);\ \ a \in \{2, 4, 6\}\ \ \text{and}\ \ b \in \{\ 2, 4, 6\}\}$

$\overline{\overline{B}} = 3 \times 3 + 3 \times 3 + 3 \times 3 = 27$

Probability of the event B: $P(B) = \frac{\overline{\overline{B}}}{\overline{\overline{\Omega}}} = \frac{27}{144} = \frac{3}{16}$

Task 7.07. (0-2) (2018 – task 15)

Two fair, six-sided dice are thrown. A is an event in which the sum of the numbers thrown is a prime number. Complete the following sentences.

(a) The sample space consists of ……….. elements.

(b) The probability of the event A is ………………………………………… .

Solution 7.07. (a) **36** (b) $\frac{\mathbf{5}}{\mathbf{12}}$

The sample space Ω (the set if all possible outcomes in the experiment):

7. Probability and statistics

$\Omega = \{(a, b);\ \ a, b \in \{1, 2, 3, 4, 5, 6\}\}$

The cardinality of the set Ω (the number of all possible outcomes):

(a) $\overline{\overline{\Omega}} = 6 \times 6 = 36$

(b) $A = \{(1,1), (1,2), (2,1), (1,4), (2,3), (3,2), (4,1),$

$(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (5,6), (6,5)\}$

$\overline{\overline{A}} = 15$

$$P(A) = \frac{15}{36} = \frac{5}{12}$$

Note: You can also use the table

+	1	2	3	4	5	6
1	**2▪**	**3▪**	**4**	**5▪**	**6**	**7▪**
2	**3▪**	**4**	**5▪**	**6**	**7▪**	**8**
3	**4**	**5▪**	**6**	**7▪**	**8**	**9**
4	**5▪**	**6**	**7▪**	**8**	**9**	**10**
5	**6**	**7▪**	**8**	**9**	**10**	**11▪**
6	**7▪**	**8**	**9**	**10**	**11▪**	**12**

Task 7.08 (0-1) (2019 – task 11)

One person is randomly selected from a class of 32 students, 18 of whom are girls. The probability that none of the girls will be selected equals:

A. $\frac{7}{9}$ **B.** $\frac{1}{32}$ **C.** $\frac{1}{14}$ **D.** $\frac{7}{16}$

Solution 7.08. D

The number of all possible outcomes is equal to 32.

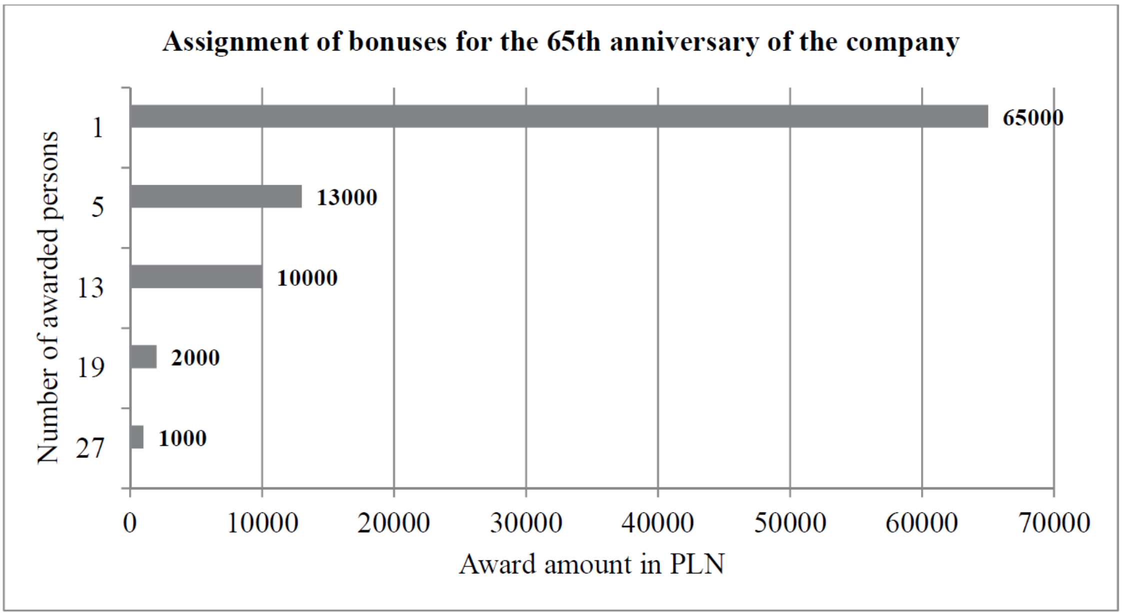
The number of boys (no girls) is equal to $32 - 18 = 14$.

The probability of the described event is equal to $\frac{14}{32}$ which is equivalent to $\frac{7}{16}$.

7. Probability and statistics

Task 7.09. (0-4) (2019 – task 14)

To celebrate its 65th anniversary, a company decided to award bonuses to 65 of its employees. The assignment of bonuses is illustrated in the diagram below.



Complete the following sentences with the correct numbers.

(a) The greatest number of employees were awarded the bonus worth PLN ....

(b) The mean of the bonuses is PLN ....

(c) The median of the awarded bonuses equals PLN ....

(d) The person who was awarded the highest bonus received ....% of the total amount allocated for all bonuses to celebrate the 65th anniversary of the company.

Solution 7.09 (a) 1000 (b) 5000 (c) 2000 (d)20%

(a) The greatest number of employees with the same bonuses value is 27.

(b) The mean m of bonuses equals

$$m = \frac{27\times1000+19\times2000+13\times10000+5\times13000+1\times65000}{27+19+13+5+1} = \frac{325000}{65} = 5000$$

(c) The median is 32nd bonus value in ordered sequence of data

$$\underbrace{\underbrace{1000,\dots\ 1000}_{27}, 2000,\dots, 2000}_{46}, 10000,\dots\ 10000, 13000,\dots, 13000, 65000$$

and this is between 27th and 46th data: PLN 2000.

(d) $\frac{65000}{325000} = 0.2 = 20\%$

Task 7.10. (0-3) (2019 – task 17)

From the set of numbers {11,12,13,14,15,16,17,18,19, 20} two numbers are randomly drawn without replacement.

Complete the following sentences with the correct numbers.

(a) The probability of drawing two numbers whose product is an odd number equals ....

(b) The probability of drawing two even numbers equals ....

(c) The probability of drawing two numbers whose difference is an odd number equals ....

Solution 7.10. (a) $\frac{2}{9}$ (b) $\frac{2}{9}$ (c) $\frac{5}{9}$

The sample space:

$\Omega = \{(x, y); x, y \in \{11,12,13,14,15,16,17,18,19, 20\}$ and x ≠ y}

$\overline{\overline{\Omega}} = 10 \times 9$

7. Probability and statistics

(a) $A = \{(x,y); x,y \in \{11,13,15,17,19\} \text{ and } x \neq y\}$

$\bar{\bar{A}} = 5 \times 4 = 20 \qquad P(A) = \frac{20}{90} = \frac{2}{9}$

(b) $B = \{(x,y); x,y \in \{12,14,16,18,20\} \text{ and } x \neq y\}$

$\bar{\bar{B}} = 5 \times 4 = 20 \qquad P(B) = \frac{20}{90} = \frac{2}{9}$

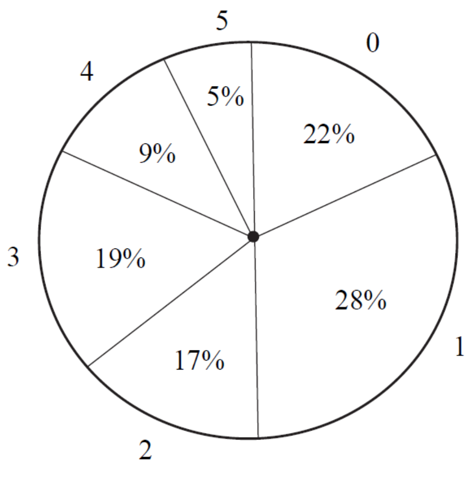
(c) $C = C_1 \cup C_2$ where

$C_1 = \{(x,y); x,y \in \{12,14,16,18,20\} \text{ and } y \in \{11,13,15,17,19\}\} \qquad \overline{\overline{C_1}} = 5 \times 5$

$C_2 = \{(x,y); x,y \in \{11,13,15,17,19,\} \text{ and } y \in \{12,14,16,18,20\}\} \qquad \overline{\overline{C_2}} = 5 \times 5$

$\bar{\bar{C}} = 5 \times 5 + 5 \times 5 = 50 \qquad P(C) = \frac{50}{90} = \frac{5}{9}$

Task 7.11 (0-1) (2020 – task 16)



The pie chart on the left shows a summary of responses given by a group of people to the question:

How many books did you read last month?

The median of the responses is:

A. 1 **B.** 1.5 **C.** 2 **D.** 2.5

Solution 7.11. B

Let’s put the data in order

$$\underbrace{\underbrace{0,\dots\dots\dots\dots,0}_{22\%},\underbrace{1,\dots\dots\dots\dots\dots,1}_{28\%}}_{50\%},\ \underbrace{\underbrace{2,\dots,2}_{17\%},\underbrace{3,\dots,3}_{19\%},4\underbrace{,\dots,4}_{9\%},\underbrace{5,\dots,5}_{5\%}}_{50\%}$$

Now we can see that the two middle data are 1 and 2, the media equals $\frac{1+2}{2}$ which is 1.5.

Task 7.12 (0-3) (2020 – task 19)

Two numbers are randomly drawn without replacement from the set {2, 3, 5, 7,11,13}. Complete the following sentences.

(a) The probability of event A in which the sum of two numbers drawn is divisible by 9 equals …………… .

(b) The probability of event B in which two odd numbers are drawn equals …………… .

(c) The probability of event C in which the product of two numbers drawn is less than 30 equals …………… .

Solution 7.12. (a) $\frac{1}{5}$ (b) $\frac{2}{3}$ (c) $\frac{7}{15}$

Let Ω be the sample space and A, B, C be the events described in (a), (b), (c) respectively.

$\Omega = \{(x, y); x, y \in \{2, 3, 5, 7, 11, 13\} \text{ and } x \neq y\}$

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$\overline{\overline{\Omega}} = 6 \times 5 = 30$

(a) $\mathrm{A} = \{(x, y); x, y \in \{2, 3, 5, 7, 11, 13\} \text{ and } \mathrm{x} \neq \mathrm{y} \text{ and } (\mathrm{x}+\mathrm{y}=9 \text{ or } \mathrm{x}+\mathrm{y}=18)\}$

$\mathrm{A} = \{(2, 7), (5, 13), (7, 2), (7, 11), (11, 7), (13, 5)\}$

$\overline{\overline{\mathrm{A}}} = 6 \qquad P(A) = \frac{6}{30} = \frac{1}{5}$

(b) $\mathrm{B} = \{(x, y); x, y \in \{3, 5, 7, 11, 13\} \text{ and } \mathrm{x} \neq \mathrm{y}\}$

$\overline{\overline{\mathrm{B}}} = 5 \times 4 = 20 \qquad P(B) = \frac{20}{30} = \frac{2}{3}$

(c) $\mathrm{C} = \{(2, 3), (2, 5), (2, 7), (2, 11), (2, 13), (3, 2), (3, 5), (3, 7), (5, 2), (5, 3), (7, 2), (7, 3), (11, 2), (13, 2)\}$

$\overline{\overline{\mathrm{C}}} = 14 \qquad P(C) = \frac{14}{30} = \frac{7}{15}$

Task 7.13. (0-3) (2021– task 15)

The number of all natural divisors of the second power of the number 2020 is equal to

A. 4 **B.** 8 **C.** 44 **D.** 45

Solution 7.13. D

Let's factorize 2020:

$$\begin{array}{r|l} 2020 & 2 \\ 1010 & 2 \\ 505 & 5 \\ 101 & 101 \\ 1 & \end{array}$$

$2020 = 2^2 \times 5 \times 101$

So the product form of the second power of 2020 is

$2020^2 = 2^4 \times 5^2 \times 101^2$

Thus, every natural divisor d of 2020^2 must be of the form

$d = 2^a \times 5^b \times 101^c$

where $a \in \{0, 1, 2, 3, 4\}$, $b \in \{0, 1, 2\}$, $c \in \{0, 1, 2\}$.

So a can be selected in 5 ways, b – in 3 ways, and c – in 3 ways.

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$5 \times 3 \times 3 = 45$

By the product rule in combinatorics, there are 45 such divisors.

Task 7.14. (0-2) (2021– task 19)

In a random experiment, two fair, distinguishable cubic dice are thrown. Let A denote an event in which the product of the values obtained is an odd number.

Complete the sentences a–b below by writing the correct numeric values in the blanks.

(a) The sample space for the experiment has .... outcomes.

(b) The probability of the event A is equal to ....

Solution 7.14. (a) 36 (b)$\frac{1}{4}$

Let Ω be the sample space.

$\Omega = \{(x, y); x, y \in \{1, 2, 3, 4, 5, 6\}\}$ $\quad \overline{\overline{\Omega}} = 6 \times 6 = 36$

The product of numbers is an odd number if and only if all factors are odd numbers, so

$\mathrm{A} = \{(x, y); x, y \in \{1, 3, 5\}\}$ $\quad \overline{\overline{\mathrm{A}}} = 3 \times 3 = 9$ $\quad P(A) = \frac{9}{36} = \frac{1}{4}$

Answers

7.01. (a) 5 (b) $\frac{1}{4}$

7.02. B

7.03. (a) 20% (b) 3

7.04. B

7.05. (a) 11.5 (b) September

7.06. (a) $\frac{11}{144}$ (b) $\frac{3}{16}$

7.07. (a) 36 (b) $\frac{5}{12}$

7.08. D

7.09. (a) 1000 (b) 5000 (c) 2000 (d)20%

7. Probability and statistics

7.10. (a) $\frac{2}{9}$ (b) $\frac{2}{9}$ (c) $\frac{5}{9}$

7.11 B

7.12. (a) $\frac{1}{5}$ (b) $\frac{2}{3}$ (c) $\frac{7}{15}$

7.13. D

7.14. (a) 36 (b) $\frac{1}{4}$