Task 8.01. (0-4) (2015– task 17)

The lateral surface of a cylinder unfolds into a square with an area of 72π . Complete the following sentences.

(a) The height of the cylinder is ..... .

(b) The radius of the cylinder's base is ..... .

(c) The area of the axial cross-section of the cylinder is ..... .

(d) The volume of the cylinder is ..... .

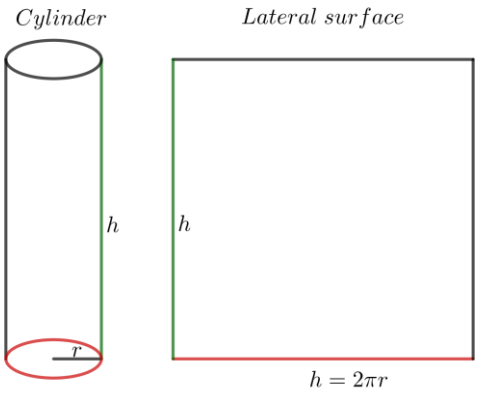
Solution 8.01. (a) $6\sqrt{2\pi}$ (b) $\frac{3\sqrt{2\pi}}{\pi}$ (c) 72 (d) $108\sqrt{2\pi}$

Let r be a radius of the of the cylinder base.

Let h be a height of the of the cylinder.

Lateral surface is a square

with the area of 72π, so $h^2 = 72\pi$.



(a) $h = 6\sqrt{2\pi}$

(b) $2\pi r = 6\sqrt{2\pi}$

$r = \frac{3\sqrt{2\pi}}{\pi}$

(c) The cross section of the cylinder is a rectangle of dimensions $2r$ and h. Crosssection Area $= 2r \times h = \frac{6\sqrt{2\pi}}{\pi} \times 6\sqrt{2\pi} = \frac{36\times 2\pi}{\pi} = 72$

(d) Let's V be the volume of the cylinder.

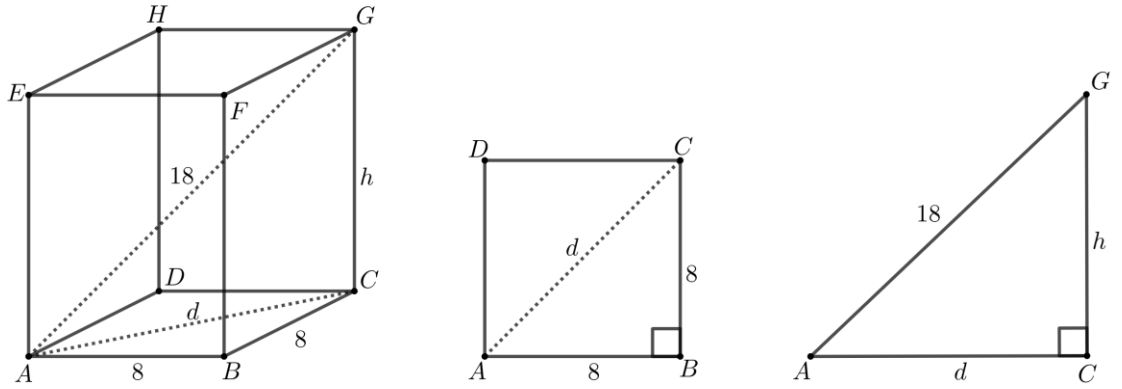
$$V = \pi r^2 h = \pi \times \left(\frac{3\sqrt{2\pi}}{\pi}\right)^2 \times 6\sqrt{2\pi} = \pi \times \frac{9\times 2\pi}{\pi^2} \times 6\sqrt{2\pi} = 108\sqrt{2\pi}$$

Task 8.02. (0-1) (2016– task 13)

In a right tetragonal prism, the length of the base edge is 8 cm. The length of the diagonal in this prism is 18 cm. The sum of all side edges of this prism equals

A. 56 cm **B.** $2\sqrt{65}$ cm **C.** 14 cm **D.** $8\sqrt{65}$ cm

Solution 8.02. A



$d = 8\sqrt{2}$

$h^2 = 18^2 - d^2 = 18^2 - \left(8\sqrt{2}\right)^2 = 324 - 128 = 196$ $h = 14$

Let S be the sum of all side (lateral) edges. $S = 4 \times 14 = 56$

Task 8.03. (0-2) (2016– task 15)

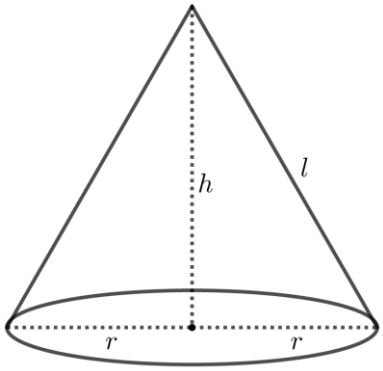
The axial cross-section of a cone is an equilateral triangle with an area of $49\sqrt{3}$.

Complete the following sentences.

(a) The area of the cone's base equals ..... .

(b) The volume of the cone equals ..... .

Solution 8.03. (a) 49π **(b) $\frac{343\pi\sqrt{3}}{2}$**



The cross section of the cone is an equilateral triangle, so $l = 2r$ and $h = r\sqrt{3}$.

The area of the triangle is $49\sqrt{3}$,

so

$\frac{1}{2} \times 2r \times r\sqrt{3} = 49\sqrt{3}$

$r = 7$

$h = 7\sqrt{3}$

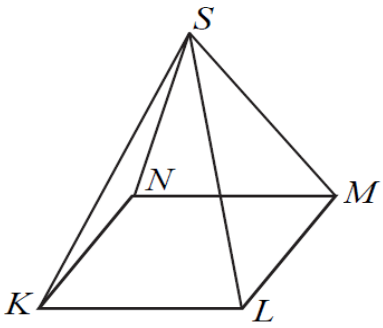
(a) Base Area $= \pi r^2 = 49\pi$

(b) $Volume = \frac{1}{2} \times 49\pi \times 7\sqrt{3} = \frac{343\pi\sqrt{3}}{2}$

Task 8.04. (0-1) (2017– task 12)

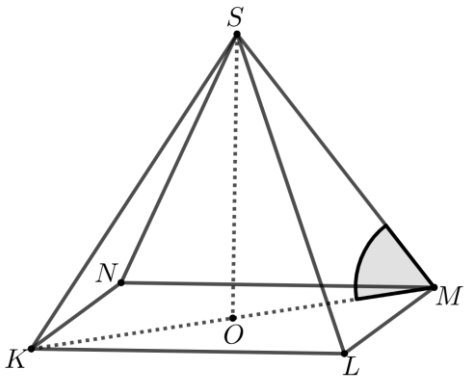
The base of a right quadrangular pyramid *KLMNS* is the square *KLMN* (see the illustration).

The angle between the lateral edge and the base of the pyramid is:



A. $\angle KLN$ **B.** $\angle KLM$ **C.** $\angle LMS$ **D.** $\angle KMS$

Solution 8.04. D



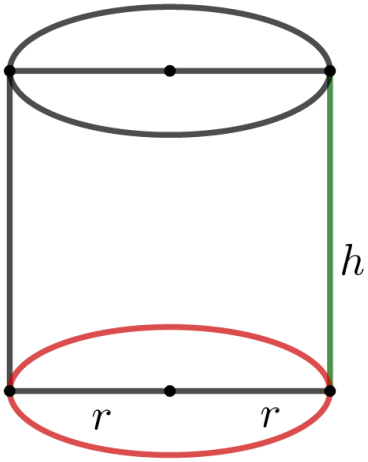
Task 8.05. (0-2) (2017– task 17)

The radius of a base of cylinder is 10. The axial cross-section of the cylinder is a square. Complete the following sentences.

(a) The total surface area of the cylinder is …… .

(b) The volume of the cylinder is …… .

Solution 8.05. (a) 600π (b) 2000π



$r = 10$

The axial cross-section of the cylinder is a square, so $h = 2r$.

$h = 20$

Let A be the total surface area.

$A = 2\pi r^2 + 2\pi rh = 200\pi + 400\pi = 600\pi$

Let V be the volume.

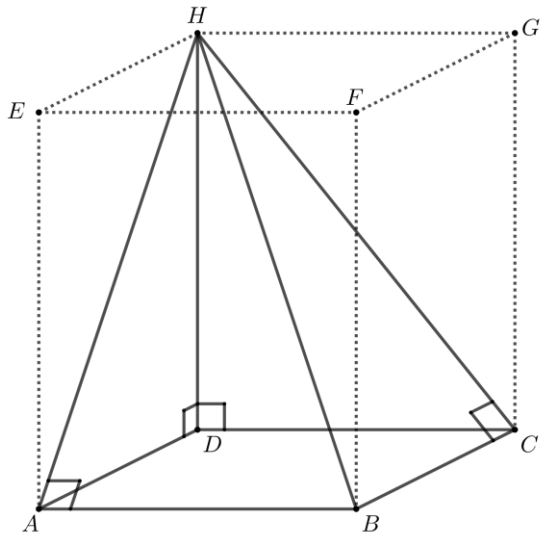
$V = \pi r^2 h = 2000\pi$

Task 8.06. (0-1) (2018– task 12)

The base of a pyramid is a rectangle, and one of the side edges of this pyramid is perpendicular to the base. The number of faces of this pyramid which are right-angled triangles is

A. 2 **B.** 1 **C.** 4 **D.** 3

Solution 8.06. C

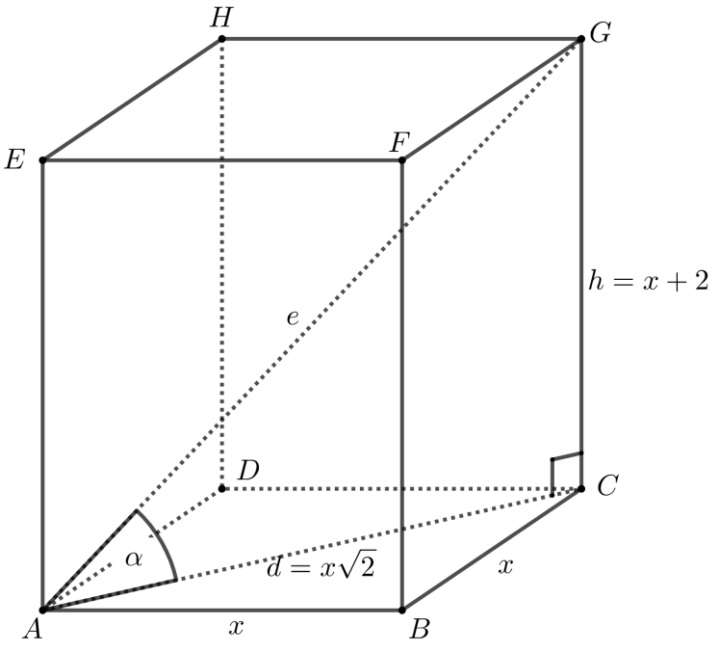


Task 8.07. (0-4) (2018– task 18)

The height of a regular quadrilateral prism is 2 units larger than the edge of the base of the prism, while the sum of the length of all edges of the prism is 92 . Complete the following sentences.

a) The height of the prism is …… .
b) The volume of the prism is …… .
c) The length of the diagonal of the prism is …… .
d) If α is the angle between the diagonal of the prism and its base, then the cosine of the angle α is …… .

Solution 8.07. (a) 9 **(b) 441** **(c) $\sqrt{179}$** **(d) $\frac{7\sqrt{358}}{179}$**



$8x + 4(x + 2) = 92$

$x = 7$

$h = 9$

$V = x^2h = 49 \times 9 = 441$

$e^2 = d^2 + h^2$

$e^2 = \left(7\sqrt{2}\right)^2 + 9^2$

$e^2 = 179$

$e = \sqrt{179}$

$\cos\alpha = \frac{d}{e}$

$\cos\alpha = \frac{7\sqrt{2}}{\sqrt{179}} = \frac{7\sqrt{358}}{179}$

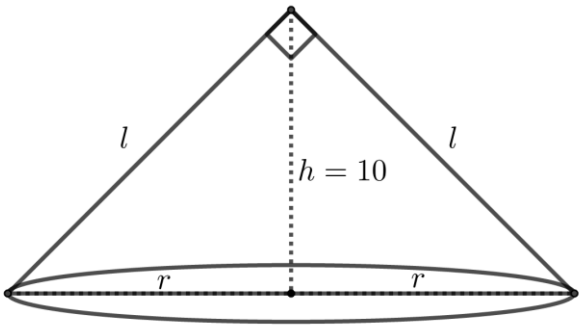
Task 8.08. (0-1) (2019– task 10)

The angle at the vertex of a cone is a right angle, and the height of the cone equals 10.

The lateral surface area of the cone is:

A. $100\pi\sqrt{3}$ **B.** $100\pi(\sqrt{2}+1)$ **C.** 100π **D.** $100\pi\sqrt{2}$

Solution 8.08. D



$r = 10$ $l = 10\sqrt{2}$

Let L be the lateral surface of the cone.

$L = \pi r l = 100\pi\sqrt{2}$

Task 8.09. (0-3) (2019 – task 15)

A rectangle with sides measuring 3 and 4 is the base of a pyramid *ABCDS*. One of the side edges of the pyramid is perpendicular to the base, while its length is the same as the length of the diagonal of the base.

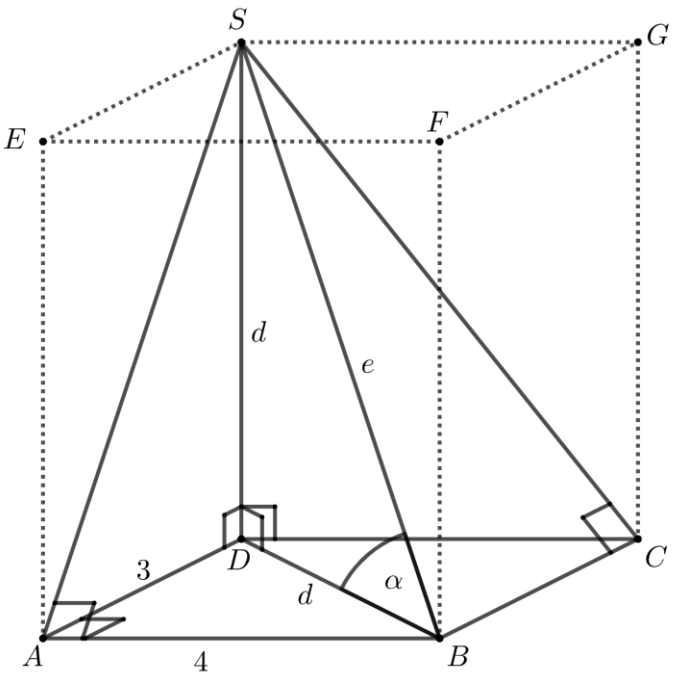
Complete the following sentences with the correct numbers.

(a) The volume of the pyramid equals …… .

(b) The angle between the longest side edge and the base of the pyramid measures ……°

(c) The pyramid has five faces, and the number of faces which are right-angled triangles equals …… .

Solution 8.09. (a) 20 (b) 45° (c) 4



The triangle ABD is right-angled

$d^2 = 3^2 + 4^2$

$d = 5$

and based on the assumptions

the triangle BDS is an isosceles right-angled triangle.

The longest lateral edge is e.

(a) Let V be the volume of the pyramid.

$$V = \frac{1}{3} \times 3 \times 4 \times d = \frac{1}{3} \times 3 \times 4 \times 5 = 20$$

(b) $\tan\alpha = \frac{d}{d} = 1 \qquad \alpha = 45°$

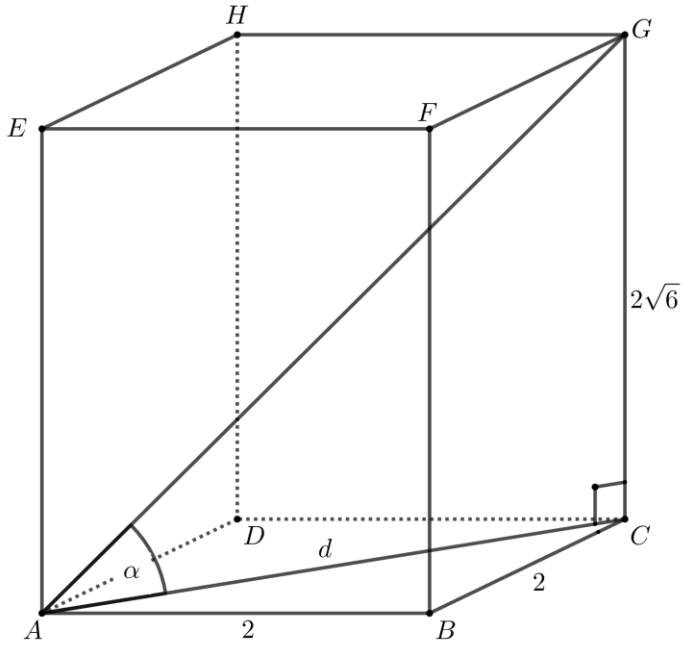
(c) There are 4 faces that are right-angled triangles: ABS, BCS, ADS, CDS.

Task 8.10. (0-1) (2020 – task 15)

In a square based prism, the base edge length is 2, and the height of the prism is $2\sqrt{6}$. The angle between the diagonal of this prism and its base is:

A. 30° **B.** 45° **C.** 60° **D.** 75°

Solution 8.10. C



$$\tan\alpha = \frac{2\sqrt{6}}{d}$$

$$d = 2\sqrt{2}$$

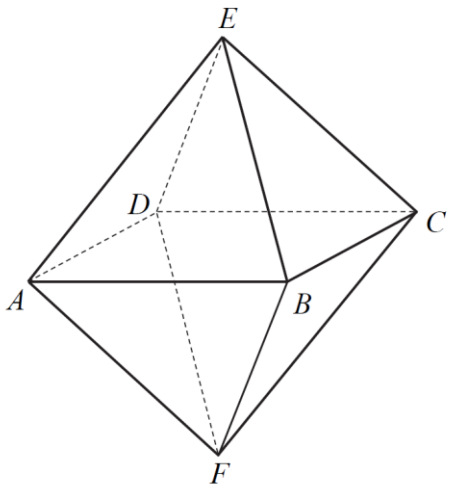
$$\tan\alpha = \frac{2\sqrt{6}}{2\sqrt{2}}$$

$$\tan\alpha = \sqrt{3}$$

$$\alpha = 60°$$

Task 8.11. (0-3) (2020 – task 21)

Two square based pyramids *ABCDE* and *ABCDF* have the same base *ABCD* (refer to the figure below). All the edges of the pyramids have the same length of 10 cm.



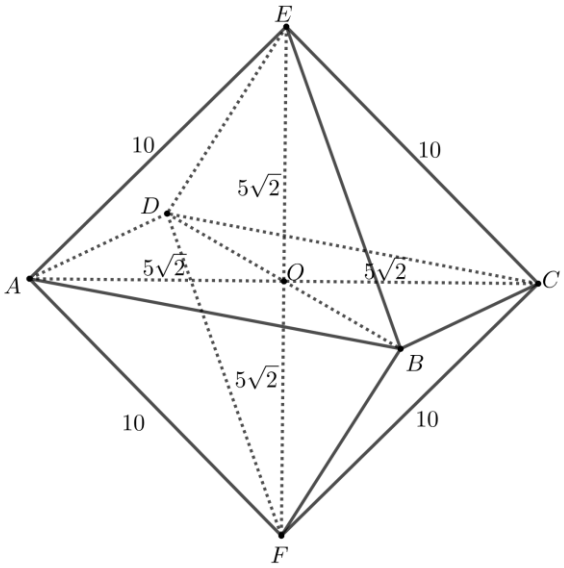
Complete the following sentences.

(a) The distance *EF* between the vertices of the pyramids equals ……
.

(b) The volume of the obtained solid is ……..
.

(c) The surface area of the obtained solid is ……..…….
.

Solution 8.11. (a) $10\sqrt{2}$ (b) $\frac{1000\sqrt{2}}{3}$ (c) $200\sqrt{3}$



(a) The quadrilateral $AFCE$ is a square with side length 10 and the segment EF is its diagonal, so $|EF| = 10\sqrt{2}$.

(b) Let V be the volume of the obtained solid (which is an octahedron).

$$V = 2 \times \frac{1}{3} \times \text{Area}(ABCD) \times |OC|$$

$$V = 2 \times \frac{1}{3} \times 100 \times 5\sqrt{2}$$

$$V = \frac{1000\sqrt{2}}{3}$$

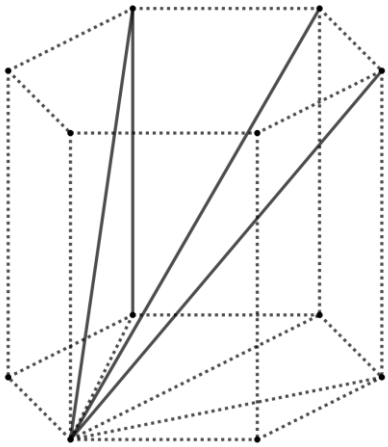
(c) Let S be the surface area of the obtained solid. The surface of the solid consists of 8 equilateral triangles with side of 10. Thus $S = 8 \times \frac{10^2\sqrt{3}}{4} = 200\sqrt{3}$.

Task 8.12. (0-1) (2021 – task 13)

A prism has 8 faces. The total number of body diagonals of this prism is equal to

A. 20 **B.** 16 **C.** 18 **D.** 40

Solution 8.12. C



Let n be the number of base edges of the prism. Then the number of all its faces is $n + 2$.

Based on the assumption $n + 2 = 8$, so $n = 6$.

This means a hexagonal prism.

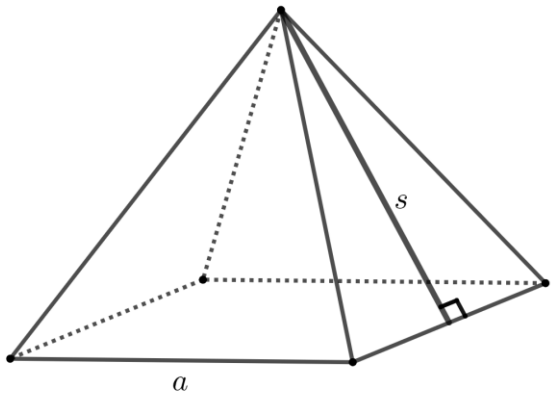
Each bottom vertex can be connect with three vertices by body diagonals. Therefore the number of all body diagonals equals $6 \times 3 = 18$.

Task 8.13. (0-1) (2021 – task 14)

In a regular square pyramid, the ratio of the total surface area to the lateral surface area equals $9:5$. Then the ratio of the length of the base edge of this pyramid to the slant height of the pyramid is equal to

A. $2:5$ **B.** $4:5$ **C.** $3:\sqrt{5}$ **D.** $8:5$

Solution 8.13. D



$$\frac{a^2+4\times\frac{1}{2}as}{4\times\frac{1}{2}as}=\frac{9}{5}$$

$$\frac{a^2+2as}{2as}=\frac{9}{5}$$

$$\frac{a+2s}{2s}=\frac{9}{5}$$

$$\frac{a}{2s}+1=\frac{9}{5}$$

$$\frac{a}{2s}=\frac{4}{5}$$

$$\frac{a}{s}=\frac{8}{5}$$

Answers

8.01. (a) $6\sqrt{2\pi}$ (b) $\frac{3\sqrt{2\pi}}{\pi}$ (c) 72 (d) $108\sqrt{2\pi}$

8.02 A

8.03. (a) 49π (b) $\frac{343\pi\sqrt{3}}{2}$

8.04. D

8.05. (a) 600π (b) 2000π

8.06. C

8.07. (a) 9 (b) 441 (c) $\sqrt{179}$ (d) $\frac{7\sqrt{358}}{179}$

8.08. D

8.09. (a) 20 (b) $45°$ (c) 4

8.10. C

8.11. (a) $10\sqrt{2}$ (b) $\frac{1000\sqrt{2}}{3}$ (c) $200\sqrt{3}$

8.12. C

8.13. D